BIG ideas

- **Variable**

  Essential Question How can you represent quantities, patterns, and relationships?

- **Properties**

  Essential Question How are properties related to algebra?

Chapter Preview

1-1 Variables and Expressions

Content Standard

A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.

Objective To write algebraic expressions

Can the number of states in the United States vary?

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

Consider the population of Florida, the area of Colorado, and the flight time from Philadelphia to San Francisco. Which of these has a value that varies? Explain.

Dynamic Activity
Using Variable Expressions

Lesson Vocabulary
- quantity
- variable
- algebraic expression
- numerical expression

A mathematical **quantity** is anything that can be measured or counted. Some quantities remain constant. Others change, or vary, and are called *variable quantities.*

Essential Understanding Algebra uses symbols to represent quantities that are unknown or that vary. You can represent mathematical phrases and real-world relationships using symbols and operations.

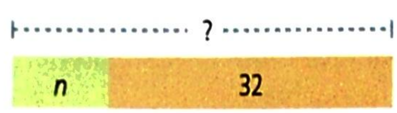
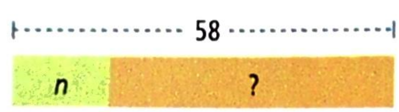
A **variable** is a symbol, usually a letter, that represents the value(s) of a variable quantity. An **algebraic expression** is a mathematical phrase that includes one or more variables. A **numerical expression** is a mathematical phrase involving numbers and operation symbols, but no variables.

Problem 1 Writing Expressions With Addition and Subtraction

Plan

How can a diagram help you write an algebraic expression?
Models like the ones shown can help you to visualize the relationships described by the word phrases.

What is an algebraic expression for the word phrase?

Word Phrase	Model	Expression
A 32 more than a number n		$n + 32$
B 58 less a number n		$58 - n$

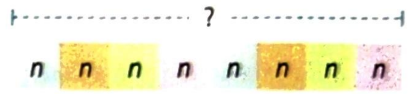
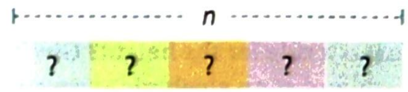
Got It? **1.** What is an algebraic expression for 18 more than a number n?

Problem 2 Writing Expressions With Multiplication and Division

Think

Is there more than one way to write an algebraic expression with multiplication? Yes. Multiplication can be represented using a dot or parentheses in addition to an ×.

What is an algebraic expression for the word phrase?

Word Phrase	Model	Expression
A 8 times a number n		$8 \times n, 8 \cdot n, 8n$
B the quotient of a number n and 5		$n \div 5, \frac{n}{5}$

Got It? **2.** What is an algebraic expression for each word phrase in parts (a) and (b)?

a. 6 times a number n

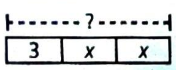
b. the quotient of 18 and a number n

c. Reasoning Do the phrases *6 less a number y* and *6 less than a number y* mean the same thing? Explain.

Problem 3 Writing Expressions With Two Operations

Plan

How can I represent the phrases visually? Draw a diagram. You can represent the phrase in Problem 2, part (A), as shown below.



What is an algebraic expression for the word phrase?

Word Phrase	Expression
A 3 more than twice a number x	$3 + 2x$
B 9 less than the quotient of 6 and a number x	$\frac{6}{x} - 9$
C the product of 4 and the sum of a number x and 7	$4(x + 7)$

Got It? **3.** What is an algebraic expression for each word phrase?

a. 8 less than the product of a number x and 4

b. twice the sum of a number x and 8

c. the quotient of 5 and the sum of 12 and a number x

In Problems 1, 2, and 3, you were given word phrases and wrote algebraic expressions. You can also translate algebraic expressions into word phrases.

Problem 4 Using Words for an Expression

Think

Is there only one way to write the expression in words? No. The operation performed on 3 and x can be described by different words like "multiply," "times," and "product."

What word phrase can you use to represent the algebraic expression $3x$?

Expression $3x$, $3 \cdot x$ — A number and a variable side by side indicate a product.

Words three times a number x or the product of 3 and a number x

Got It? **4.** What word phrase can you use to represent the algebraic expression?

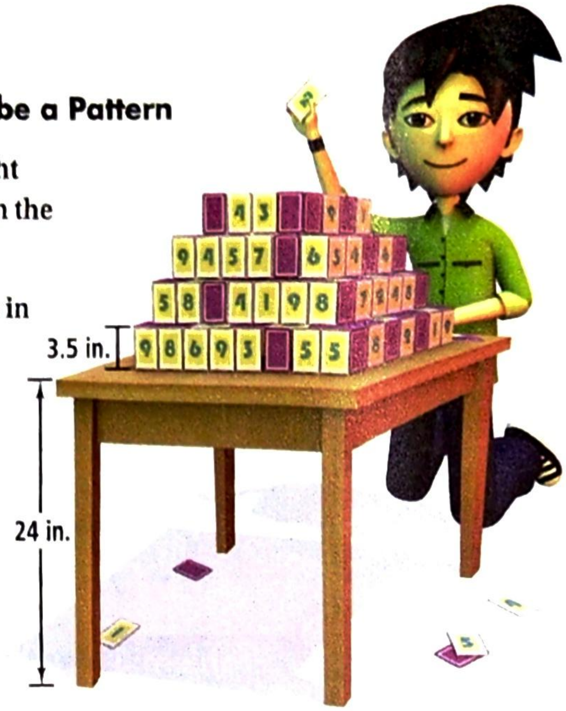
a. $x + 8.1$ **b.** $10x + 9$ **c.** $\frac{n}{3}$ **d.** $5x - 1$

You can use words or an algebraic expression to write a mathematical rule that describes a real-life pattern.

Problem 5 Writing a Rule to Describe a Pattern

Hobbies The table below shows how the height above the floor of a house of cards depends on the number of levels.

A What is a rule for the height? Give the rule in words and as an algebraic expression.



House of Cards

Number of Levels	Height (in.)
2	$(3.5 \cdot 2) + 24$
3	$(3.5 \cdot 3) + 24$
4	$(3.5 \cdot 4) + 24$
n	?

Know
Numerical expressions for the height given several different numbers of levels

Need
A rule for finding the height given a house with n levels

Plan
Look for a pattern in the table. Describe the pattern in words. Then use the words to write an algebraic expression.

Rule in Words Multiply the number of levels by 3.5 and add 24.

Rule as an Algebraic Expression The variable n represents the number of levels in the house of cards.

$3.5n + 24$ This expression lets you find the height for n levels.

B **A group of students built another house of cards that had 10 levels. Each card was 4 inches tall, and the height from the floor to the top of the house of cards was 70 inches. How tall would the house of cards be if they built an 11th level?**

Since each card was 4 inches tall, adding 1 more level would increase the total height of the house of cards by 4 inches.

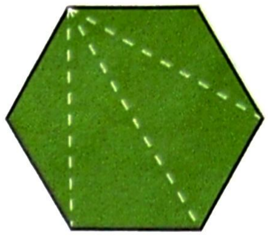
The house of cards would be $70 + 4$, or 74 inches tall if the 11th level were added.

C **Another group of students built a third house of cards with n levels. Each card was 5 inches tall, and the height from the floor to the top of the house of cards was $34 + 5n$ inches. How tall would the house of cards be if the group added 1 more level of cards?**

Since each card was 5 inches tall, adding 1 more level would increase the total height of the house of cards by 5 inches.

The house of cards would be $34 + 5n + 5$ in. tall if the next level were added.

Got It? **5.** Suppose you draw a segment from any one vertex of a regular polygon to the other vertices. A sample for a regular hexagon is shown below. Use the table to find a pattern. What is a rule for the number of nonoverlapping triangles formed? Give the rule in words and as an algebraic expression.



Triangles in Polygons

Number of Sides of Polygon	Number of Triangles
4	4 − 2
5	5 − 2
6	6 − 2
n	■

Lesson Check

Do you know HOW?

1. Is each expression *algebraic* or *numerical*?

a. $7 \div 2$ **b.** $4m + 6$ **c.** $2(5 - 4)$

2. What is an algebraic expression for each phrase?

a. the product of 9 and a number t

b. the difference of a number x and $\frac{1}{2}$

c. the sum of a number m and 7.1

d. the quotient of 207 and a number n

Use words to describe each algebraic expression.

3. $6c$ **4.** $x - 1$

5. $\frac{t}{2}$ **6.** $3t - 4$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. Vocabulary Explain the difference between numerical expressions and algebraic expressions.

8. Reasoning Use the table to decide whether $49n + 0.75$ or $49 + 0.75n$ represents the total cost to rent a truck that you drive n miles.

Truck Rental Fees

Number of Miles	Cost
1	\$49 + (\$.75 × 1)
2	\$49 + (\$.75 × 2)
3	\$49 + (\$.75 × 3)
n	■

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice Write an algebraic expression for each word phrase. See Problems 1–3.

9. 4 more than p
10. y minus 12
11. the quotient of n and 8
12. the product of 15 and c
13. a number t divided by 82
14. the sum of 13 and twice a number h
15. 6.7 more than the product of 5 and n
16. 9.85 less than the product of 37 and t

Write a word phrase for each algebraic expression. See Problem 4.

17. $q + 5$ **18.** $\frac{y}{5}$ **19.** $12x$ **20.** $49 + m$

21. $9n + 1$ **22.** $\frac{z}{8} - 9$ **23.** $15 - \frac{1.5}{d}$ **24.** $2(5 - n)$

Write a rule in words and as an algebraic expression to model the relationship in each table.

See Problem 5.

25. Sightseeing While on vacation, you rent a bicycle. You pay \$9 for each hour you use it. It costs \$5 to rent a helmet while you use the bicycle.

Bike Rental

Number of Hours	Rental Cost
1	$(\$9 \times 1) + \$5$
2	$(\$9 \times 2) + \$5$
3	$(\$9 \times 3) + \$5$
n	■

26. Sales At a shoe store, a salesperson earns a weekly salary of \$150. A salesperson is also paid \$2.00 for each pair of shoes he or she sells during the week.

Shoe Sales

Pairs of Shoes Sold	Total Earned
5	$\$150 + (\$2 \times 5)$
10	$\$150 + (\$2 \times 10)$
15	$\$150 + (\$2 \times 15)$
n	■

B Apply

Write an algebraic expression for each word phrase.

27. 8 minus the product of 9 and r

28. the sum of 15 and x, plus 7

29. 4 less than three sevenths of y

30. the quotient of 12 and the product of 5 and t

31. Error Analysis A student writes the word phrase "the quotient of n and 5" to describe the expression $\frac{5}{n}$. Describe and correct the student's error.

32. Think About a Plan The table at the right shows the number of bagels a shop gives you per "baker's dozen." Write an algebraic expression that gives the rule for finding the number of bagels in any number b of baker's dozens.

- What is the pattern of increase in the number of bagels?
- What operation can you perform on b to find the number of bagels?

Bagels

Baker's Dozens	Number of Bagels
1	13
2	26
3	39
b	■

33. Tickets You and some friends are going to a museum. Each ticket costs \$4.50.

a. If n is the number of tickets purchased, write an expression that gives the total cost of buying n tickets.

b. Suppose the total cost for n tickets is \$36. What is the total cost if one more ticket is purchased?

34. Volunteering Serena and Tyler are wrapping gift boxes at the same pace. Serena starts first, as shown in the diagram. Write an algebraic expression that represents the number of boxes Tyler will have wrapped when Serena has wrapped x boxes.

35. Multiple Choice Which expression gives the value in dollars of d dimes?

Ⓐ $0.10d$ Ⓑ $0.10 + d$ Ⓒ $\frac{0.10}{d}$ Ⓓ $10d$

Open-Ended **Describe a real-world situation that each expression might model. Tell what each variable represents.**

36. $5t$ **37.** $b + 3$ **38.** $\frac{40}{h}$

Challenge

39. Reasoning You write $(5 - 2) \div n$ to represent the phrase *2 less than 5 divided by a number n*. Your friend writes $(5 \div n) - 2$. Are these both reasonable interpretations? Can verbal descriptions lack precision? Explain.

Write two different expressions that could both represent the given diagram.

40.

x	1	1	1	1
x	1	1	1	1
x	1	1	1	1

41.

x	1	1	1	1
x	1	1		

Standardized Test Prep

SAT/ACT

42. What is an algebraic expression for *2 less than the product of 3 and a number x*?

Ⓐ $3x - 2$ Ⓑ $(3 - 2)x$ Ⓒ $3 - 2x$ Ⓓ $2 - 3x$

43. Which word phrase can you use to represent the algebraic expression $n \div 8$?

Ⓕ the product of a number n and 8
Ⓗ the difference of a number n and 8
Ⓖ the quotient of a number n and 8
Ⓘ the quotient of 8 and a number n

44. A state park charges an entrance fee plus \$18 for each night of camping. The table shows this relationship. Which algebraic expression describes the total cost of camping for n nights?

Ⓐ $20n + 18$ Ⓒ $18n + 20n$
Ⓑ $18n + 20$ Ⓓ $18n - 20$

Camping

Nights	Total Cost
1	(\$18 × 1) + \$20
2	(\$18 × 2) + \$20
3	(\$18 × 3) + \$20
n	■

Mixed Review

Find each sum or difference. Write each answer in simplest form. See p. 803.

45. $\frac{1}{4} + \frac{1}{2}$ **46.** $\frac{9}{14} - \frac{2}{7}$ **47.** $\frac{2}{5} + \frac{3}{10}$ **48.** $\frac{5}{6} - \frac{2}{3}$

Get Ready! **To prepare for Lesson 1-2, do Exercises 49–52.**

Find the greatest common factor of each pair of numbers. See p. 799.

49. 3 and 6 **50.** 12 and 15 **51.** 7 and 11 **52.** 8 and 12

1-2 Order of Operations and Evaluating Expressions

Content Standard

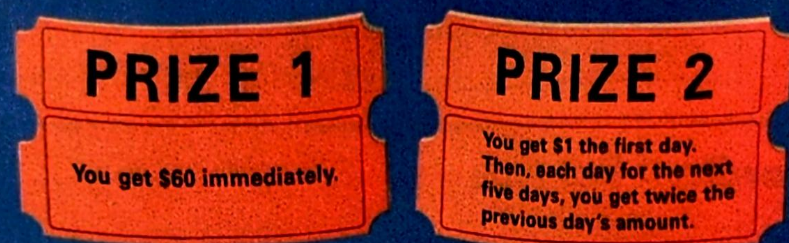
A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.

Objectives To simplify expressions involving exponents
To use the order of operations to evaluate expressions

SOLVE IT! **Getting Ready!**

MATHEMATICAL PRACTICES

You've won! For a door prize, you get to choose between the two options shown. Which is the better prize? Why?



Dynamic Activity Order of Operations

Lesson Vocabulary
- power
- exponent
- base
- simplify
- evaluate

Essential Understanding You can use *powers* to shorten how you represent repeated multiplication, such as $2 \times 2 \times 2 \times 2 \times 2 \times 2$.

A **power** has two parts, a *base* and an *exponent*. The **exponent** tells you how many times to use the **base** as a factor. You read the power 2^3 as "two to the third power" or "two cubed." You read 5^2 as "five to the fifth power" or "five squared."

base, exponent

$$\underbrace{2^3}_{\text{power}} = 2 \cdot 2 \cdot 2$$

You **simplify** a numerical expression when you replace it with its single numerical value. For example, the simplest form of $2 \cdot 8$ is 16. To simplify a power, you replace it with its simplest name.

Problem 1 Simplifying Powers

Think
What does the exponent indicate? It shows the number of times you use the base as a factor.

What is the simplified form of the expression?

A 10^7

$$10^7 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$
$$= 10{,}000{,}000$$

B $(0.2)^5$

$$(0.2)^5 = 0.2 \cdot 0.2 \cdot 0.2 \cdot 0.2 \cdot 0.2$$
$$= 0.00032$$

Got It? **1.** What is the simplified form of each expression?

a. 3^4 **b.** $\left(\frac{2}{3}\right)^3$ **c.** $(0.5)^3$

Essential Understanding When simplifying an expression, you need to perform operations in the correct order.

You might think about simplifying the expression $2 + 3 \times 5$ in two ways:

Add first.

$2 + 3 \times 5 = 5 \times 5 = 25$ ✗

Multiply first.

$2 + 3 \times 5 = 2 + 15 = 17$ ✔

Both results may seem sensible, but only the second result is considered correct. This is because the second way uses the order of operations that mathematicians have agreed to follow. Always use the following order of operations:

take note

Key Concept Order of Operations

1. Perform any operation(s) inside grouping symbols, such as parentheses ( ) and brackets [ ]. A fraction bar also acts as a grouping symbol.
2. Simplify powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Problem 2 Simplifying a Numerical Expression

Think

How do you simplify an expression that contains a fraction? You start by simplifying the numerator and denominator. Then you divide the numerator by the denominator.

What is the simplified form of each expression?

A $(6 - 2)^3 \div 2$

$$\begin{aligned} (6 - 2)^3 \div 2 &= 4^3 \div 2 && \text{Subtract inside parentheses.} \\ &= 64 \div 2 && \text{Simplify the power.} \\ &= 32 && \text{Divide.} \end{aligned}$$

B $\frac{2^4 - 1}{5}$

$$\begin{aligned} \frac{2^4 - 1}{5} &= \frac{16 - 1}{5} && \text{Simplify the power.} \\ &= \frac{15}{5} && \text{Subtract.} \\ &= 3 && \text{Divide.} \end{aligned}$$

Got It? **2.** What is the simplified form of each expression?

a. $5 \cdot 7 - 4^2 \div 2$

b. $12 - 25 \div 5$

c. $\frac{4 + 3^4}{7 - 2}$

d. Reasoning How does a fraction bar act as a grouping symbol? Explain.

When two or more variables, or a number and variables, are written together, treat them as if they were within parentheses. So $4xy$ is equivalent to $(4xy)$, and $xy^2 = (xy^2)$. You **evaluate** an algebraic expression by replacing each variable with a given number. Then simplify the expression using the order of operations.

Problem 3 Evaluating Algebraic Expressions

Plan

How is this Problem like ones you've seen before?
You begin by substituting numbers for the variables. After substituting, you have numerical expressions just like the ones in Problem 2.

What is the value of the expression for $x = 5$ and $y = 2$?

A $x^2 + x - 12 \div y^2$

$x^2 + x - 12 \div y^2 = 5^2 + 5 - 12 \div 2^2$	Substitute 5 for x and 2 for y.
$= 25 + 5 - 12 \div 4$	Simplify powers.
$= 25 + 5 - 3$	Divide.
$= 27$	Add and subtract from left to right.

B $(xy)^2 \div (xy)$

$(xy)^2 \div xy = (5 \cdot 2)^2 \div (5 \cdot 2)$	Substitute 5 for x and 2 for y.
$= 10^2 \div 10$	Multiply inside parentheses.
$= 100 \div 10$	Simplify the power.
$= 10$	Divide.

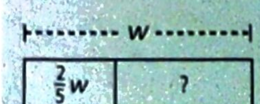
Got It? **3.** What is the value of each expression when $a = 3$ and $b = 4$ in parts (a)-(b)?

a. $3b - a^2$ **b.** $2b^2 - 7a$

Problem 4 Evaluating a Real-World Expression

Think

How can a model help you write the expression?
This model shows that spending money equals your wages w minus the amount you save: $\frac{2}{5}w$.



Banking **What is an expression for the spending money you have left after depositing $\frac{2}{5}$ of your wages in savings? Evaluate the expression for weekly wages of \$40, \$50, \$75, and \$100.**

Know
- Savings equals $\frac{2}{5}$ of wages.
- Various weekly wages

Need
- Expression for spending money
- Amount of spending money for various weekly wages

Plan
Write an algebraic expression and evaluate it for each amount of weekly wages. Use a table to organize your results.

Relate spending money equals wages minus $\frac{2}{5}$ of wages

Define Let w = your wages.

Write $w \quad - \quad \frac{2}{5} \quad \cdot \quad w$

The expression $w - \frac{2}{5} \cdot w$ represents the amount of money you have left after depositing $\frac{2}{5}$ of your wages in savings.

Spending Money

Wages (w)	$w - \frac{2}{5}w$	Total Spending Money (\$)
40	$40 - \frac{2}{5}(40)$	24
50	$50 - \frac{2}{5}(50)$	30
75	$75 - \frac{2}{5}(75)$	45
100	$100 - \frac{2}{5}(100)$	60

Got It? 4. The shipping cost for an order at an online store is $\frac{1}{10}$ the cost of the items you order. What is an expression for the total cost of a given order? What are the total costs for orders of \$43, \$79, \$95, and \$103?

Lesson Check

Do you know HOW?

What is the simplified form of each expression?

1. 5^2 **2.** 2^3 **3.** $\left(\frac{3}{4}\right)^2$

Evaluate each expression for $x = 3$ and $y = 4$.

4. $x^2 + 2(x + y)$

5. $(xy)^3$

6. $4x^2 - 3xy$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. Vocabulary Identify the exponent and the base in 4^3.

8. Error Analysis A student simplifies an expression as shown below. Find the error and simplify the expression correctly.

$$\begin{aligned} 23 - 8 \cdot 2 + 3^2 &= 23 - 8 \cdot 2 + 9 \\ &= 15 \cdot 2 + 9 \\ &= 30 + 9 \\ &= 39 \ \text{✗} \end{aligned}$$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Simplify each expression. See Problems 1 and 2.

9. 3^5 **10.** 4^3 **11.** 2^4 **12.** 10^8

13. $\left(\frac{2}{3}\right)^3$ **14.** $\left(\frac{1}{2}\right)^4$ **15.** $(0.4)^6$ **16.** 7^4

17. $20 - 2 \cdot 3^2$ **18.** $6 + 4 \div 2 + 3$ **19.** $(6^2 - 3^3) \div 2$ **20.** $5 \cdot 2^2 \div 2 + 8$

21. $80 - (4 - 1)^3$ **22.** $52 + 8^2 - 3(4 - 2)^3$ **23.** $\frac{6^4 \div 3^2}{9}$ **24.** $\frac{2 \cdot 7 + 4}{9 \div 3}$

Evaluate each expression for $s = 4$ and $t = 8$. See Problem 3.

25. $(s + t)^3$ **26.** $s^4 + t^2 + s \div 2$ **27.** $(st)^2 \div (st^2)$

28. $3st^2 \div (st) + 6$ **29.** $(t - s)^5$ **30.** $(2s)^2 t$

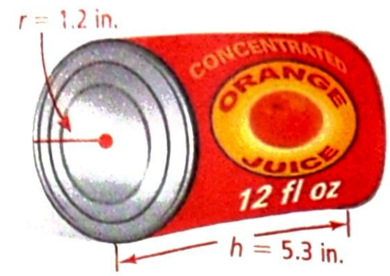
31. $2st^2 - s^2$ **32.** $2s^2 - t^3 \div 16$ **33.** $\frac{(3s)^3 t + t}{s}$

34. Write an expression for the amount of change you will get when you pay for a purchase p with a \$20 bill. Make a table to find the amounts of change you will get for purchases of \$11.59, \$17.50, \$19.00, and \$20.00. See Problem 4.

35. An object's momentum is defined as the product of its mass m and velocity v. Write an expression for the momentum of an object. Make a table to find the momentums of a vehicle with a mass of 1000 kg moving at a velocity of 15 m/s, 20 m/s, and 25 m/s.

B Apply

36. Geometry The expression $\pi r^2 h$ represents the volume of a cylinder with radius r and height h.



a. What is the volume, to the nearest tenth of a cubic inch, of the juice can at the right? Use 3.14 for π.

b. Reasoning About how many cubic inches, to the nearest tenth of a cubic inch, does a fluid ounce of juice fill?

Simplify each expression.

37. $2[(8 - 4)^5 \div 8]$

38. $3[(4 - 2)^5 - 20]$

39. $10 - (2^3 + 4) \div 3 - 1$

40. $\dfrac{22 + 1^3 + (3^4 - 7^2)}{2^3}$

41. $3[42 - 2(10^2 - 9^2)]$

42. $\dfrac{2[8 + (67 - 2^6)^3]}{9}$

43. Think About a Plan The snack bar at your school has added sushi to its menu. The ingredients for one roll include sushi rice, seaweed sheets, cucumbers, cream cheese, and 3 oz of smoked salmon. One roll can be cut into 8 servings. Write an expression for the amount of salmon needed to make s servings of sushi. How much salmon is needed to make 16 servings? 24 servings? 80 servings? 100 servings?

- What operations are needed in your calculations?
- Use a table to help you organize your results. What will you use for the column headings in your table?

44. Salary You earn $10 for each hour you work at a canoe rental shop. Write an expression for your salary for working the number of hours h. Make a table to find how much you earn for working 10 h, 20 h, 30 h, and 40 h.

Evaluate each expression for the given values of the variables.

45. $3(s - t)^2$; $s = 4, t = 1$

46. $2x - y^2$; $x = 7, y = 3.5$

47. $3m^2 - n$; $m = 2, n = 6$

48. $(2a + 2b)^2$; $a = 3, b = 4$

49. $2p^2 + (2q)^2$; $p = 4, q = 3$

50. $(4c - d + 0.2)^2 - 10c$; $c = 3.1, d = 4.6$

51. $\dfrac{3g + 6}{h}$; $g = 5, h = 7$

52. $\dfrac{2w + 3v}{v^2}$; $v = 6, w = 1$

53. Writing Consider the expression $(1 + 5)^2 - (18 \div 3)$. Can you perform the operations in different orders and still get the correct answer? Explain.

54. A student wrote the expressions shown and claimed they were equal for all values of x and y.

$(x + y)^2$

$x^2 + y^2$

a. Evaluate each expression for $x = 1$ and $y = 0$.

b. Evaluate each expression for $x = 1$ and $y = 2$.

c. Open-Ended Choose another pair of values for x and y. Evaluate each expression for those values.

d. Writing Is the student's claim correct? Justify your answer.

55. Find the value of $14 + 5 \cdot 3 - 3^2$. Then change two operation signs so that the value of the expression is 8.

Challenge Use grouping symbols to make each equation true.

56. $9 + 3 - 2 + 4 = 6$

57. $16 - 4 \div 2 + 3 = 9$

58. $4^2 - 5 \cdot 2 + 1 = 1$

59. $3 \cdot 4 + 5 - 6 + 7 = 28$

60. a. Geometry A cone has a slant height ℓ of 11 cm and a radius r of 3 cm. Use the expression $\pi r(\ell + r)$ to find the surface area of the cone. Use 3.14 for π. Round to the nearest tenth of a square centimeter.

b. Reasoning Does the surface area of the cone double if the radius doubles? If the slant height doubles? Explain.

Standardized Test Prep

SAT/ACT

61. What is the simplified form of $4 + 10 \div 4 + 6$?

(A) 1.4 (B) 9.5 (C) 12.5 (D) 24

62. What is the value of $(2a)^2b - 2c^2$ for $a = 2$, $b = 4$, and $c = 3$?

(F) 14 (G) 28 (H) 32 (I) 46

63. A shirt is on sale for $25 at the local department store. The sales tax equals $\frac{1}{25}$ of the shirt's price. What is the total cost of the shirt including sales tax?

(A) $17 (B) $26 (C) $27 (D) $33

64. You can find the distance in feet that an object falls in t seconds using the expression $16t^2$. If you drop a ball from a tall building, how far does the ball fall in 3 s?

(F) 16 ft (G) 48 ft (H) 96 ft (I) 144 ft

Mixed Review

Write an algebraic expression for each word phrase. See Lesson 1-1.

65. 4 more than p

66. 5 minus the product of y and 3

67. the quotient of m and 10

68. 3 times the difference of 7 and d

Tell whether each number is *prime* or *composite*. See p. 798.

69. 17 **70.** 33 **71.** 43 **72.** 91

Get Ready! To prepare for Lesson 1-3, do Exercises 73–80.

Write each fraction as a decimal and each decimal as a fraction. See p. 802.

73. $\frac{3}{5}$ **74.** $\frac{7}{8}$ **75.** $\frac{2}{3}$ **76.** $\frac{4}{7}$

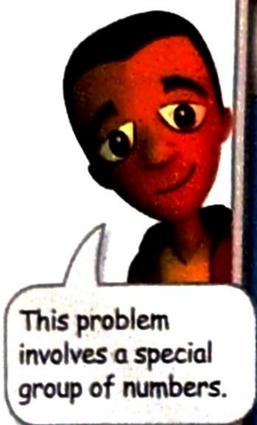
77. 0.7 **78.** 0.07 **79.** 4.25 **80.** 0.425

1-3 Real Numbers and the Number Line

Content Standard

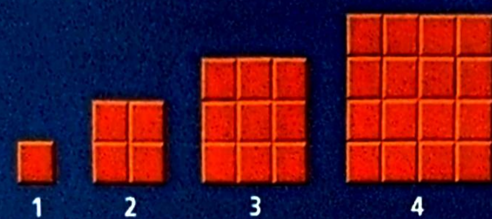
Prepares for N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

Objectives To classify, graph, and compare real numbers
To find and estimate square roots



SOLVE IT! Getting Ready!

If the pattern continues, which will be the first figure to contain more than 200 square units? Explain your reasoning.



MATHEMATICAL PRACTICES

Dynamic Activity
Exploring Square Roots
The Real Number Line

Lesson Vocabulary
- square root
- radicand
- radical
- perfect square
- set
- element of a set
- subset
- rational numbers
- natural numbers
- whole numbers
- integers
- irrational numbers
- real numbers
- inequality

The diagrams in the Solve It model what happens when you multiply a number by itself to form a product. When you do this, the original number is called a *square root* of the product.

take note

Key Concept Square Root

Algebra A number a is a **square root** of a number b if $a^2 = b$.

Example $7^2 = 49$, so 7 is a square root of 49.

Essential Understanding You can use the definition above to find the exact square roots of some nonnegative numbers. You can approximate the square roots of other nonnegative numbers.

The radical symbol $\sqrt{}$ indicates a nonnegative square root, also called a *principal square root*. The expression under the radical symbol is called the **radicand.**

$$\text{radical symbol} \rightarrow \sqrt{a} \leftarrow \text{radicand}$$

Together, the radical symbol and radicand form a **radical.** You will learn about negative square roots in Lesson 1-6.

Problem 1 Simplifying Square Root Expressions

Think

How can you find a square root?
Find a number that you can multiply by itself to get a product that is equal to the radicand.

What is the simplified form of each expression?

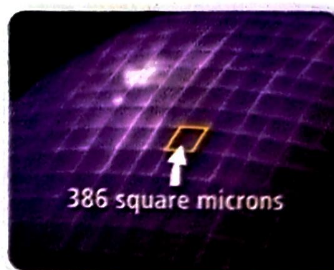
A $\sqrt{81} = 9$ $9^2 = 81$, so 9 is a square root of 81.

B $\sqrt{\frac{9}{16}} = \frac{3}{4}$ $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$, so $\frac{3}{4}$ is a square root of $\frac{9}{16}$.

Got It? **1.** What is the simplified form of each expression?

a. $\sqrt{64}$ **b.** $\sqrt{25}$ **c.** $\sqrt{\frac{1}{36}}$ **d.** $\sqrt{\frac{81}{121}}$

The square of an integer is called a **perfect square.** For example, 49 is a perfect square because $7^2 = 49$. When a radicand is not a perfect square, you can estimate the square root of the radicand.

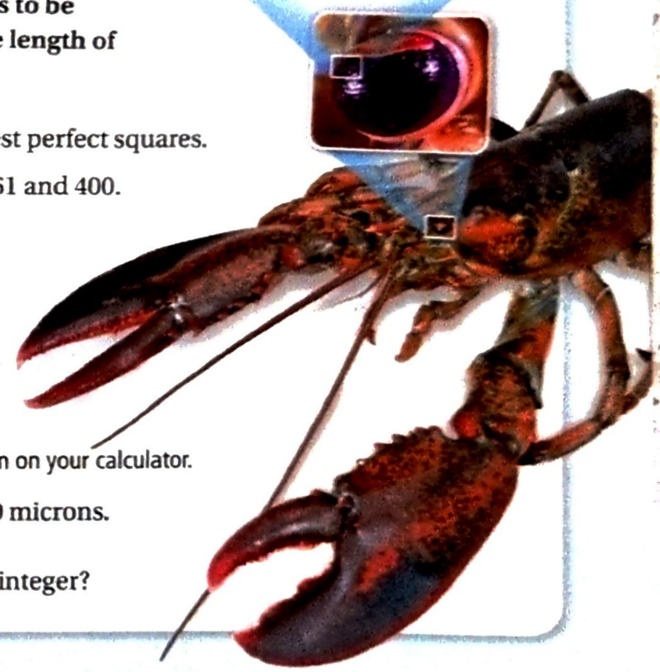


Problem 2 Estimating a Square Root STEM

Biology **Lobster eyes are made of tiny square regions. Under a microscope, the surface of the eye looks like graph paper. A scientist measures the area of one of the squares to be 386 square microns. What is the approximate side length of the square to the nearest micron?**

Plan

How can you get started?
The square root of the area of a square is equal to its side length. So, find $\sqrt{386}$.



Method 1 Estimate $\sqrt{386}$ by finding the two closest perfect squares.

The perfect squares closest to 386 are 361 and 400.

$19^2 = 361$

$20^2 = 400 \quad \longleftarrow 386$

Since 386 is closer to 400, $\sqrt{386} \approx 20$, and the side length is about 20 microns.

Method 2 Estimate $\sqrt{386}$ using a calculator.

$\sqrt{386} \approx 19.6$ Use the square root function on your calculator.

The side length of the square is about 20 microns.

Got It? **2.** What is the value of $\sqrt{34}$ to the nearest integer?

Essential Understanding Numbers can be classified by their characteristics. Some types of numbers can be represented on the number line.

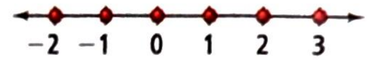
You can classify numbers using *sets*. A **set** is a well-defined collection of objects. Each object is called an **element of the set.** A **subset** of a set consists of elements from the given set. You can list the elements of a set within braces { }.

A **rational number** is any number that you can write in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. A rational number in decimal form is either a terminating decimal such as 5.45 or a repeating decimal such as 0.41666 . . . , which you can write as $0.41\overline{6}$. Each graph below shows a subset of the rational numbers on a number line.

Natural numbers $\{1, 2, 3, \ldots\}$

Whole numbers $\{0, 1, 2, 3, \ldots\}$

Integers $\{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$



An **irrational number** cannot be represented as the quotient of two integers. In decimal form, irrational numbers do not terminate or repeat. Here are some examples.

$$0.1010010001\ldots \qquad \pi = 3.14159265\ldots$$

Some square roots are rational numbers and some are irrational numbers. If a whole number is not a perfect square, its square root is irrational.

Rational $\sqrt{4} = 2$ $\sqrt{25} = 5$

Irrational $\sqrt{3} = 1.73205080\ldots$ $\sqrt{10} = 3.16227766\ldots$

Rational numbers and irrational numbers form the set of **real numbers.**

Problem 3 Classifying Real Numbers

Think

What clues can you use to classify real numbers?

Look for negative signs, fractions, decimals that do or do not terminate or repeat, and radicands that are not perfect squares.

To which subsets of the real numbers does each number belong?

A 15 natural numbers, whole numbers, integers, rational numbers

B -1.4583 rational numbers (since -1.4583 is a terminating decimal)

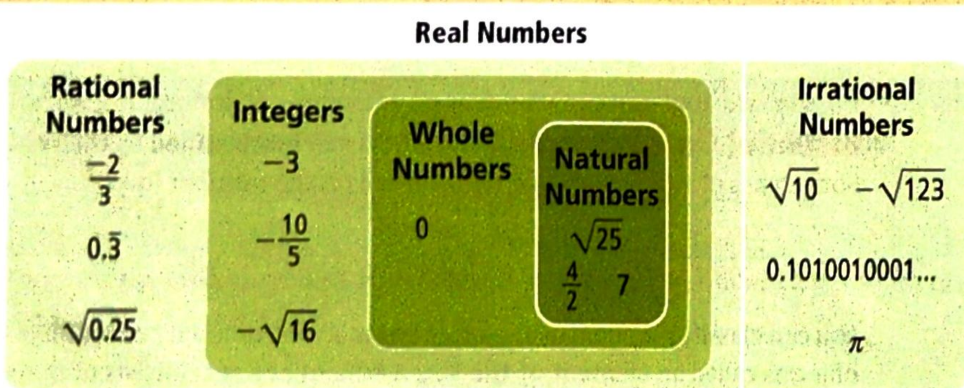
C $\sqrt{57}$ irrational numbers (since 57 is not a perfect square)

Got It? 3. To which subsets of the real numbers does each number belong?

a. $\sqrt{9}$ b. $\frac{3}{10}$ c. -0.45 d. $\sqrt{12}$

take note

Concept Summary Real Numbers



An **inequality** is a mathematical sentence that compares the values of two expressions using an inequality symbol. The symbols are

$<$, less than	$\leq$, less than or equal to
$>$, greater than	$\geq$, greater than or equal to

Plan

How can you compare numbers? Write the numbers in the same form, such as decimal form.

Problem 4 Comparing Real Numbers

What is an inequality that compares the numbers $\sqrt{17}$ and $4\frac{1}{3}$?

$\sqrt{17} = 4.12310\ldots$ Write the square root as a decimal.

$4\frac{1}{3} = 4.\overline{3}$ Write the fraction as a decimal.

$\sqrt{17} < 4\frac{1}{3}$ Compare using an inequality symbol.

Got It? 4. a. What is an inequality that compares the numbers $\sqrt{129}$ and 11.52?

b. Reasoning In Problem 4, is there another inequality you can write that compares the two numbers? Explain.

You can graph and order all real numbers using a number line.

Problem 5 Graphing and Ordering Real Numbers

Multiple Choice **What is the order of $\sqrt{4}$, 0.4, $-\frac{2}{3}$, $\sqrt{2}$, and -1.5 from least to greatest?**

Ⓐ $-\frac{2}{3}$, 0.4, -1.5, $\sqrt{2}$, $\sqrt{4}$

Ⓑ -1.5, $\sqrt{2}$, 0.4, $\sqrt{4}$, $-\frac{2}{3}$

Ⓒ -1.5, $-\frac{2}{3}$, 0.4, $\sqrt{2}$, $\sqrt{4}$

Ⓓ $\sqrt{4}$, $\sqrt{2}$, 0.4, $-\frac{2}{3}$, -1.5

Know
Five real numbers

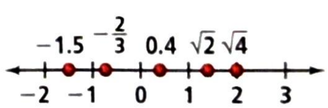
Need
Order of numbers from least to greatest

Plan
Graph the numbers on a number line.

Think

Why is it useful to rewrite numbers in decimal form? It allows you to compare numbers whose values are close, like $\frac{1}{4}$ and 0.26.

First, write the numbers that are not in decimal form as decimals: $\sqrt{4} = 2$, $-\frac{2}{3} \approx -0.67$, and $\sqrt{2} \approx 1.41$. Then graph all five numbers on the number line to order the numbers, and read the graph from left to right.



From least to greatest, the numbers are -1.5, $-\frac{2}{3}$, 0.4, $\sqrt{2}$, and $\sqrt{4}$. The correct answer is C.

Got It? 5. Graph 3.5, -2.1, $\sqrt{9}$, $-\frac{7}{2}$, and $\sqrt{5}$ on a number line. What is the order of the numbers from least to greatest?

Lesson Check

Do you know HOW?

Name the subset(s) of the real numbers to which each number belongs.

1. $\sqrt{11}$

2. -7

3. Order $\frac{47}{10}$, 4.1, -5, and $\sqrt{16}$ from least to greatest.

4. A square card has an area of 15 in.^{2}. What is the approximate side length of the card?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary What are the two subsets of the real numbers that form the set of real numbers?

6. Vocabulary Give an example of a rational number that is not an integer.

Reasoning Tell whether each square root is *rational* or *irrational*. Explain.

7. $\sqrt{100}$

8. $\sqrt{0.29}$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Simplify each expression. See Problem 1.

9. $\sqrt{36}$ **10.** $\sqrt{169}$ **11.** $\sqrt{16}$ **12.** $\sqrt{900}$ **13.** $\sqrt{\frac{36}{49}}$

14. $\sqrt{\frac{25}{81}}$ **15.** $\sqrt{\frac{1}{9}}$ **16.** $\sqrt{\frac{121}{16}}$ **17.** $\sqrt{1.96}$ **18.** $\sqrt{0.25}$

Estimate the square root. Round to the nearest integer. See Problem 2.

19. $\sqrt{17}$ **20.** $\sqrt{35}$ **21.** $\sqrt{242}$ **22.** $\sqrt{61}$ **23.** $\sqrt{320}$

Find the approximate side length of each square figure to the nearest whole unit.

24. a mural with an area of 18 m^2

25. a game board with an area of 160 in.^{2}

26. a helicopter launching pad with an area of 3000 ft^2

Name the subset(s) of the real numbers to which each number belongs See Problem 3.

27. $\frac{2}{3}$ **28.** 13 **29.** -1 **30.** $-\frac{19}{100}$ **31.** π

32. -2.38 **33.** $\frac{17}{4573}$ **34.** $\sqrt{144}$ **35.** $\sqrt{113}$ **36.** $\frac{59}{2}$

Compare the numbers in each exercise using an inequality symbol. See Problem 4.

37. $5\frac{2}{3}, \sqrt{29}$ **38.** $-3.1, -\frac{16}{5}$ **39.** $\frac{4}{3}, \sqrt{2}$ **40.** $9.6, \sqrt{96}$

41. $-\frac{7}{11}, -0.63$ **42.** $\sqrt{115}$, 10.72104 ... **43.** $-\frac{22}{25}, -0.\overline{8}$ **44.** $\sqrt{184}$, 15.56987 ...

Order the numbers in each exercise from least to greatest. See Problem 5.

45. $\frac{1}{2}, -2, \sqrt{5}, -\frac{7}{4}, 2.4$ **46.** $-3, \sqrt{31}, \sqrt{11}, 5.5, -\frac{60}{11}$ **47.** $-6, \sqrt{20}, 4.3, -\frac{59}{9}$

48. $\frac{10}{3}, 3, \sqrt{8}, 2.9, \sqrt{7}$ **49.** $-\frac{13}{6}, -2.1, -\frac{26}{13}, -\frac{9}{4}$ **50.** $-\frac{1}{6}, -0.3, \sqrt{1}, -\frac{2}{13}, \frac{7}{8}$

B Apply

51. **Think About a Plan** A stage designer paid \$4 per square foot for flooring to be used in a square room. If the designer spent \$600 on the flooring, about how long is a side of the room? Round to the nearest foot.
   - How is the area of a square related to its side length?
   - How can you estimate the length of a side of a square?

Tell whether each statement is *true* or *false*. Explain.

52. All negative numbers are integers.

53. All integers are rational numbers.

54. All square roots are irrational numbers.

55. No positive number is an integer.

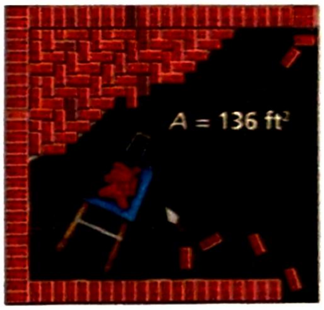
56. **Reasoning** A restaurant owner is going to panel a square portion of the restaurant's ceiling. The portion to be paneled has an area of 185 ft^2. The owner plans to use square tin ceiling panels with a side length of 2 ft. What is the first step in finding out whether the owner will be able to use a whole number of panels?

Show that each number is rational by writing it in the form $\frac{a}{b}$, where a and b are integers.

57. 417 58. 0.37 59. 2.01 60. 2.1 61. 3.06

62. **Error Analysis** A student says that $\sqrt{7}$ is a rational number because you can write $\sqrt{7}$ as the quotient $\frac{\sqrt{7}}{1}$. Is the student correct? Explain.

STEM 63. **Construction** A contractor is tiling a square patio that has the area shown at the right. What is the approximate side length of the patio? Round to the nearest foot.



64. **Open-Ended** You are tutoring a younger student. How would you explain rational numbers, irrational numbers, and how they are different?

65. **Geometry** The irrational number π, equal to 3.14159 . . ., is the ratio of a circle's circumference to its diameter. In the sixth century, the mathematician Brahmagupta estimated the value of π to be $\sqrt{10}$. In the thirteenth century, the mathematician Fibonacci estimated the value of π to be $\frac{864}{275}$. Which is the better estimate? Explain.

66. **Home Improvement** If you lean a ladder against a wall, the length of the ladder should be $\sqrt{(x)^2 + (4x)^2}$ ft to be considered safe. The distance x is how far the ladder's base is from the wall. Estimate the desired length of the ladder when the base is positioned 5 ft from the wall. Round your answer to the nearest tenth.

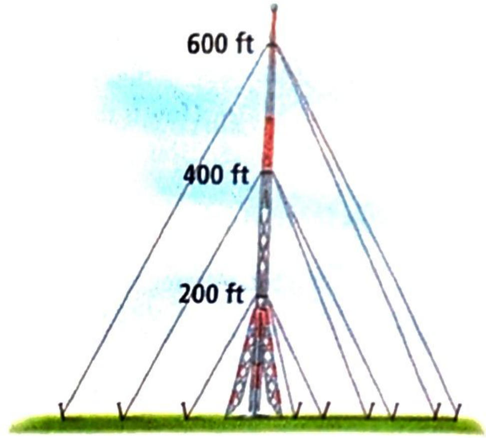
67. **Writing** Is there a greatest integer on the real number line? A least fraction? Explain.

68. **Reasoning** Choose three intervals on the real number line that contain both rational and irrational numbers. Do you think that any given interval on the real number line contains both rational and irrational numbers? Explain.

Challenge

69. **Reasoning** Sometimes the product of two positive numbers is less than either number. Describe the numbers for which this is true.

STEM 70. **Antennas** Guy wires are attached to an antenna tower at the heights h shown at the right. Use the expression $\sqrt{h^2 + (0.55h)^2}$ to estimate the wire length for each height. If three wires are attached at each height, what is the minimum total amount of wire needed?



71. **Cube Roots** The number a is the *cube root* of a number b if $a^3 = b$. For example, the cube root of 8 is 2 because $2^3 = 8$. Find the cube root of each number.

a. 64 **b.** 1000 **c.** 343 **d.** 2197

Standardized Test Prep

SAT/ACT

72. A square picture has an area of 225 in.^{2}. What is the side length of the picture?

(A) 5 in. (B) 15 in. (C) 25 in. (D) 225 in.

73. To simplify the expression $9 \cdot (33 - 5^2) \div 2$, what do you do first?

(F) Divide by 2. (G) Subtract 5. (H) Multiply by 9. (I) Square 5.

74. The table at the right shows the number of pages you can read per minute. Which algebraic expression gives a rule for finding the number of pages read in any number of minutes m?

(A) m (C) $2m$

(B) $m + 2$ (D) $\frac{m}{2}$

Reading

Minutes	Pages Read
1	2
2	4
3	6
m	■

Mixed Review

Evaluate each expression for the given values of the variables. See Lesson 1-2.

75. $(r - t)^2; r = 11, t = 7$ 76. $3m^2 + n; m = 5, n = 3$ 77. $(2x)^2y; x = 4, y = 8$

Write an algebraic expression for each word phrase. See Lesson 1-1.

78. the sum of 14 and x

79. 4 multiplied by the sum of y and 1

80. 3880 divided by z

81. the product of t and the quotient of 19 and 3

Get Ready! To prepare for Lesson 1-4, do Exercises 82–85.

Simplify each expression. See Lesson 1-2.

82. $4 + 7 \cdot 2$ 83. $(7 + 1)9$ 84. $2 + 22 \cdot 20$ 85. $6 + 18 \div 6$

1-4 Properties of Real Numbers

Content Standard

Prepares for N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

Objective To identify and use properties of real numbers

SOLVE IT! **Getting Ready!**

Tell whether each pair of expressions is equal by completing each statement with = or ≠. Explain your answers.

$34 + 12 \; \underline{?} \; 12 + 34$	$18 \div \frac{1}{18} \; \underline{?} \; 1$
$100 - 1 \; \underline{?} \; 1 - 100$	$45 - 1 \; \underline{?} \; 45$
$0 + 180 \; \underline{?} \; 180$	$6 \times \frac{1}{6} \; \underline{?} \; 1$

MATHEMATICAL PRACTICES The Solve It illustrates numerical relationships that are always true for real numbers.

Lesson Vocabulary
- equivalent expressions
- deductive reasoning
- counterexample

Essential Understanding Relationships that are always true for real numbers are called *properties,* which are rules used to rewrite and compare expressions.

Two algebraic expressions are **equivalent expressions** if they have the same value for all values of the variable(s). The following properties show expressions that are equivalent for all real numbers.

take note

Properties   Properties of Real Numbers

Let a, b, and c be any real numbers.

Commutative Properties of Addition and Multiplication

Changing the order of the addends does not change the sum. Changing the order of the factors does not change the product.

	Algebra	Example
Addition	$a + b = b + a$	$18 + 54 = 54 + 18$
Multiplication	$a \cdot b = b \cdot a$	$12 \cdot \frac{1}{2} = \frac{1}{2} \cdot 12$

Associative Properties of Addition and Multiplication

Changing the grouping of the addends does not change the sum. Changing the grouping of the factors does not change the product.

Addition	$(a + b) + c = a + (b + c)$	$(23 + 9) + 4 = 23 + (9 + 4)$
Multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(7 \cdot 9) \cdot 10 = 7 \cdot (9 \cdot 10)$

take note

Properties Properties of Real Numbers

Let a be any real number.

Identity Properties of Addition and Multiplication

The sum of any real number and 0 is the original number. The product of any real number and 1 is the original number.

	Algebra	Example
Addition	$a + 0 = a$	$5\frac{3}{4} + 0 = 5\frac{3}{4}$
Multiplication	$a \cdot 1 = a$	$67 \cdot 1 = 67$

Zero Property of Multiplication

The product of a and 0 is 0.	$a \cdot 0 = 0$	$18 \cdot 0 = 0$

Multiplication Property of -1

The product of -1 and a is $-a$.	$-1 \cdot a = -a$	$-1 \cdot 9 = -9$

Problem 1 Identifying Properties

Think

What math symbols give you clues about the properties?
Parentheses, operation symbols, and the numbers 0 and 1 may indicate certain properties.

What property is illustrated by each statement?

A $42 \cdot 0 = 0$ Zero Property of Multiplication

B $(y + 2.5) + 28 = y + (2.5 + 28)$ Associative Property of Addition

C $10x + 0 = 10x$ Identity Property of Addition

Got It? **1.** What property is illustrated by each statement?

a. $4x \cdot 1 = 4x$ **b.** $x + (\sqrt{y} + z) = x + (z + \sqrt{y})$

You can use properties to help you solve some problems using mental math.

Problem 2 Using Properties for Mental Calculations

Plan

How can you make the addition easier?
Look for numbers having decimal parts you can add easily, such as 0.75 and 0.25.

Movies **A movie ticket costs \$7.75. A drink costs \$2.40. Popcorn costs \$1.25. What is the total cost for a ticket, a drink, and popcorn? Use mental math.**

$(7.75 + 2.40) + 1.25 = (2.40 + 7.75) + 1.25$	Commutative Property of Addition
$= 2.40 + (7.75 + 1.25)$	Associative Property of Addition
$= 2.40 + 9$	Simplify inside parentheses.
$= 11.40$	Add.

The total cost is \$11.40.

Got It? **2.** A can holds 3 tennis balls. A box holds 4 cans. A case holds 6 boxes. How many tennis balls are in 10 cases? Use mental math.

Problem 3 Writing Equivalent Expressions

Simplify each expression.

A $5(3n)$

Know	Need	Plan
An expression	Groups of numbers that can be simplified	Use properties to group or reorder parts of the expression.

$5(3n) = (5 \cdot 3)n$ Associative Property of Multiplication

$= 15n$ Simplify.

B $(4 + 7b) + 8$

$(4 + 7b) + 8 = (7b + 4) + 8$ Commutative Property of Addition

$= 7b + (4 + 8)$ Associative Property of Addition

$= 7b + 12$ Simplify.

C $\frac{6xy}{y}$

$\frac{6xy}{y} = \frac{6x \cdot y}{1 \cdot y}$ Rewrite denominator using Identity Property of Multiplication.

$= \frac{6x}{1} \cdot \frac{y}{y}$ Use rule for multiplying fractions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

$= 6x \cdot 1$ $x \div 1 = x$ and $y \div y = 1$.

$= 6x$ Identity Property of Multiplication

Got It? **3.** Simplify each expression.

a. $2.1(4.5x)$ **b.** $6 + (4h + 3)$ **c.** $\frac{8m}{12mn}$

In Problem 3, reasoning and properties were used to show that two expressions are equivalent. This is an example of *deductive reasoning*. **Deductive reasoning** is the process of reasoning logically from given facts to a conclusion.

To show that a statement is *not* true, find an example for which it is not true. An example showing that a statement is false is a **counterexample.** You need only one counterexample to prove that a statement is false.

Problem 4 Using Deductive Reasoning and Counterexamples

Plan
Look for a counterexample to show the statement is false. If you don't find one, try to use properties to show that it is true.

Is the statement *true* or *false*? If it is false, give a counterexample.

A **For all real numbers a and b, $a \cdot b = b + a$.**

False. $5 \cdot 3 \neq 3 + 5$ is a counterexample.

B **For all real numbers a, b, and c, $(a + b) + c = b + (a + c)$.**

True. Use properties of real numbers to show that the expressions are equivalent.

$(a + b) + c = (b + a) + c$ Commutative Property of Addition

$= b + (a + c)$ Associative Property of Addition

Got It? **4. Reasoning** Is each statement in parts (a) and (b) *true* or *false*? If it is false, give a counterexample. If true, use properties of real numbers to show the expressions are equivalent.

a. For all real numbers j and k, $j \cdot k = (k + 0) \cdot j$.

b. For all real numbers m and n, $m(n + 1) = mn + 1$.

c. Is the statement in part (A) of Problem 4 false for *every* pair of real numbers a and b? Explain.

Lesson Check

Do you know HOW?

Name the property that each statement illustrates.

1. $x + 12 = 12 + x$
2. $5 \cdot (12 \cdot x) = (5 \cdot 12) \cdot x$
3. You buy a sandwich for \$2.95, an apple for \$.45, and a bottle of juice for \$1.05. What is the total cost?
4. Simplify $\frac{24cd}{c}$.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary Tell whether the expressions in each pair are equivalent.

a. $5x \cdot 1$ and $1 + 5x$

b. $1 + (2t + 1)$ and $2 + 2t$

6. Justify each step.

$$\begin{aligned} 3 \cdot (10 \cdot 12) &= 3 \cdot (12 \cdot 10) \\ &= (3 \cdot 12) \cdot 10 \\ &= 36 \cdot 10 \\ &= 360 \end{aligned}$$

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

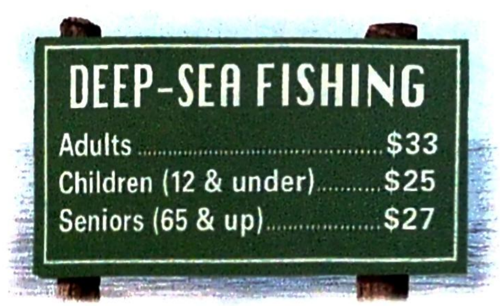
Name the property that each statement illustrates. See Problem 1.

7. $75 + 6 = 6 + 75$
8. $\frac{7}{9} \cdot 1 = \frac{7}{9}$
9. $h + 0 = h$
10. $389 \cdot 0 = 0$
11. $27 \cdot \pi = \pi \cdot 27$
12. $9 \cdot (-1 \cdot x) = 9 \cdot (-x)$

Mental Math **Simplify each expression.** See Problem 2.

13. $21 + 6 + 9$
14. $10 \cdot 2 \cdot 19 \cdot 5$
15. $0.1 + 3.7 + 5.9$
16. $4 \cdot 5 \cdot 13 \cdot 5$
17. $55.3 + 0.2 + 23.8 + 0.7$
18. $0.25 \cdot 12 \cdot 4$

19. **Fishing Trip** The sign at the right shows the costs for a deep-sea fishing trip. How much will the total cost be for 1 adult, 2 children, and 1 senior citizen to go on a fishing trip? Use mental math.



Simplify each expression. Justify each step.

See Problem 3.

20. $8 + (9t + 4)$ **21.** $9(2x)$ **22.** $(4 + 105x) + 5$ **23.** $(10p)11$

24. $(12 \cdot r) \cdot 13$ **25.** $(2 + 3x) + 9$ **26.** $4 \cdot (x \cdot 6.3)$ **27.** $1.1 + (7d + 0.1)$

28. $\frac{56ab}{b}$ **29.** $\frac{1.5mn}{m}$ **30.** $\frac{13p}{pq}$ **31.** $\frac{33xy}{3x}$

Use deductive reasoning to tell whether each statement is *true* or *false*. If it is false, give a counterexample. If true, use properties of real numbers to show the expressions are equivalent.

See Problem 4.

32. For all real numbers r, s, and t, $(r \cdot s) \cdot t = t \cdot (s \cdot r)$.

33. For all real numbers p and q, $p \div q = q \div p$.

34. For all real numbers x, $x + 0 = 0$.

35. For all real numbers a and b, $-a \cdot b = a \cdot (-b)$.

B Apply

36. Error Analysis Your friend shows you the problem at the right. He says that the Associative Property allows you to change the order in which you complete two operations. Is your friend correct? Explain.

$$\cancel{(5 \cdot 11) + 9 = 5 \cdot (11 + 9)}$$
$$\cancel{= 5 \cdot 20}$$
$$\cancel{= 100}$$

37. Travel It is 258 mi from Tulsa, Oklahoma, to Dallas, Texas. It is 239 mi from Dallas, Texas, to Houston, Texas.

a. What is the total distance of a trip from Tulsa to Dallas to Houston?

b. What is the total distance of a trip from Houston to Dallas to Tulsa?

c. Explain how you can tell whether the distances described in parts (a) and (b) are equal by using reasoning.

Tell whether the expressions in each pair are equivalent.

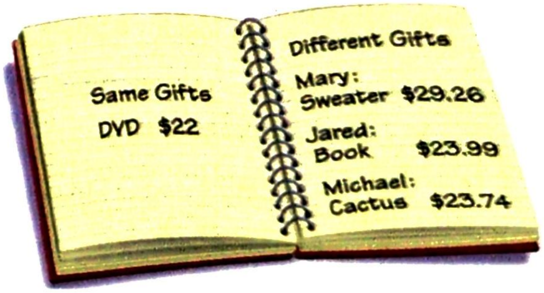
38. $2 + h + 4$ and $2 \cdot h \cdot 4$ **39.** $9y \cdot 0$ and 1 **40.** $3x$ and $3x \cdot 1$

41. $m(1 - 1)$ and 0 **42.** $(9 - 7) + \pi$ and 2π **43.** $(3 + 7) + m$ and $m + 10$

44. $\frac{63ab}{7a}$ and $9ab$ **45.** $\frac{11x}{(2 + 5 - 7)}$ and $11x$ **46.** $\frac{7t}{4 - 8 + \sqrt{9}}$ and $7t$

47. Think About a Plan Hannah makes a list of possible gifts for Mary, Jared, and Michael. She has two plans and can spend a total of $75 for all gifts. Which plan(s) can Hannah afford?

- What property can you use to make it easier to find the total cost of different gifts?
- What number do you compare to the total cost of each plan to decide whether it is affordable?



48. Writing Suppose you are mixing red and blue paint in a bucket. Do you think the final color of the mixed paint will be the same whether you add the blue paint or the red paint to the bucket first? Relate your answer to a property of real numbers.

Simplify each expression. Justify each step.

49. $25 \cdot 3.9 \cdot 4$ **50.** $(4.4 \div 4.4)(x + 7)$ **51.** $(7^6 - 6^5)(8 - 8)$

Reasoning **Answer each question. Use examples to justify your answers.**

52. Is subtraction commutative?

53. Is subtraction associative?

54. Is division commutative?

55. Is division associative?

Challenge

56. Patterns The Commutative Property of Addition lets you rewrite addition expressions. How many different ways can you write $a + b + c$? Show each way.

57. Reasoning Suppose you know that $a(b + c) = ab + ac$ is true for all real numbers a, b, and c. Use the properties of real numbers to prove that $(b + c)a = ba + ca$ is true for all real numbers a, b, and c.

Standardized Test Prep

SAT/ACT

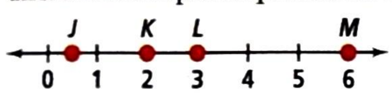
58. What is the simplified form of $(1.2 + 0) + 4.6 + 3.8$?

Ⓐ 1.2 Ⓑ 8.0 Ⓒ 8.4 Ⓓ 9.6

59. Which expression is equal to $3 \cdot 3 \cdot 8 \cdot 8 \cdot 3$?

Ⓕ $3 \cdot 8$ Ⓖ 3^8 Ⓗ $3^3 \cdot 8^2$ Ⓘ $3 \cdot 3 + 2 \cdot 8$

60. There are four points plotted on the number line below.



Which expression represents the greatest amount?

Ⓐ $M \div L$ Ⓑ $M - L$ Ⓒ $J + K$ Ⓓ $L - K$

61. Lane 1 at your local track is 0.25 mi long. You live 0.5 mi away from the track. Which of the following results in the shortest jog?

Ⓕ jogging 6 times around the track in Lane 1

Ⓖ jogging to the track and then 5 times around the track in Lane 1

Ⓗ jogging to the track, 3 times around the track in Lane 1, and then home

Ⓘ jogging 8 times around the track in Lane 1

Mixed Review

Order the numbers in each exercise from least to greatest. See Lesson 1-3.

62. $-6, 6^3, 1.6, \sqrt{6}$ **63.** $\frac{8}{5}, 1.4, -17, 10^2$ **64.** $1.75, -4.5, \sqrt{4}, 14^1$

Get Ready! **To prepare for Lesson 1-5, do Exercises 65–68.**

Find each sum or difference. See p. 803.

65. $3 + 11$ **66.** $\frac{3}{8} + \frac{5}{8}$ **67.** $9.7 - 8.6$ **68.** $\frac{5}{9} - \frac{5}{10}$

1 Mid-Chapter Quiz

MathXL® for School
Go to PowerAlgebra.com

Do you know HOW?

Write an algebraic expression for each phrase.

1. a number n divided by 4

2. 2 less than the product of 5 and n

3. The table shows how the total cost of a field trip depends on the number of students. What is a rule for the total cost of the tickets? Give the rule in words and as an algebraic expression.

Field Trip

Number of Students	Total Cost
20	$(12 \cdot 20) + 150$
40	$(12 \cdot 40) + 150$
60	$(12 \cdot 60) + 150$

4. The sign shows the costs associated with a whitewater rafting trip. Write an expression to determine the cost of 3 children and 1 adult renting equipment for a whitewater rafting trip that lasts h hours.

Whitewater Tours

Adult Ticket	\$53
Child Ticket	\$32
Equipment Rental	\$5 per hour

Simplify each expression.

5. $24 \div (3 + 2^2)$

6. $\sqrt{144}$

Evaluate each expression for the given values of the variables.

7. $3x \cdot 2 \div y$; $x = 3$ and $y = 6$

8. $(4a)^3 \div (b - 2)$; $a = 2$ and $b = 4$

9. Name the subset(s) of real numbers to which each number belongs. Then order the numbers from least to greatest.
$\sqrt{105}$, -4, $\frac{4}{3}$

10. Estimate $\sqrt{14}$ to the nearest integer.

11. What property is shown in the following equation?

$$(5 + 8) + 11 = 5 + (8 + 11)$$

12. Use the table below. If the total cost for n sandwiches is \$16.50, what is the total cost when 1 more sandwich is bought?

Lunch Menu

Salad	\$6.25
Sandwich	\$5.50
Drink	\$2.75

Do you UNDERSTAND?

13. What word phrases represent the expressions $-2 + 3x$ and $3x + (-2)$? Are the two expressions equivalent? Explain.

14. Use grouping symbols to make the following equation true.

$$4^2 + 2 \cdot 3 = 54$$

15. Choose the correct word to complete the following sentence: A natural number is (*always, sometimes, never*) a whole number.

16. How many natural numbers are in the set of numbers from -10 to 10 inclusive? Explain.

17. What is the simplified form of $\frac{3abc}{abc}$, when $abc \neq 0$? Explain using the properties of real numbers.

18. Reasoning Are the associative properties true for all integers? Explain.

19. Use the Commutative Property of Multiplication to rewrite the expression $(x \cdot y) \cdot z$ in two different ways.

1-5 Adding and Subtracting Real Numbers

Content Standard

Prepares for N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

Objective To find sums and differences of real numbers

You may find using a number line helpful here.

MATHEMATICAL PRACTICES

SOLVE IT!

Getting Ready!

You have kept track of the activity on a gift card, as shown at the right. The values are negative (red) when you spend money and positive (black) when you add money.

You want to give the card to a friend. How much money must you add to make the card worth $25? Explain your reasoning.

9/3 get gift card	$50
9/4 buy new game	$19
9/7 buy new jacket	$29
9/10 Aunt Sue adds $	$25
9/13 buy new headphones	$13
need to add to be $25	?

Lesson Vocabulary
- absolute value
- opposites
- additive inverses

Essential Understanding You can add or subtract any real numbers using a number line model. You can also add or subtract real numbers using rules involving absolute value.

Problem 1 Using Number Line Models

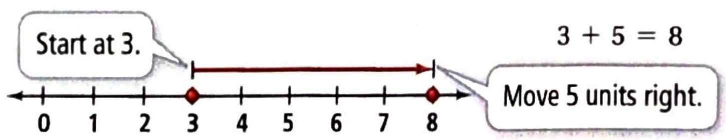
Think

How do you know which direction to move along the number line?

If the number added is positive, move to the right. If the number added is negative, move to the left.

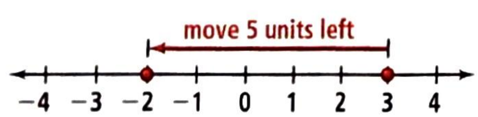
What is each sum? Use a number line.

A $3 + 5$



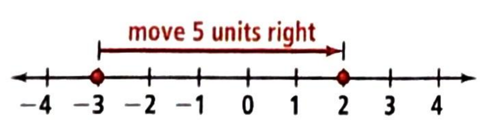
$3 + 5 = 8$

B $3 + (-5)$



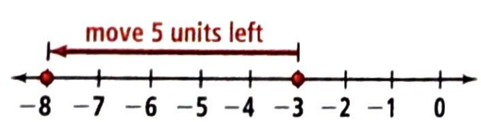
$3 + (-5) = -2$

C $-3 + 5$



$-3 + 5 = 2$

D $-3 + (-5)$

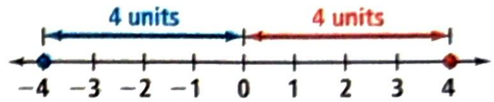


$-3 + (-5) = -8$

Got It? 1. What is $-8 + 4$? Use a number line.

The **absolute value** of a number is its distance from 0 on a number line. Absolute value is always nonnegative since distance is always nonnegative.

For example, the absolute value of 4 is 4 and the absolute value of -4 is 4. You can write this as $|4| = 4$ and $|-4| = 4$.



You can use absolute value when you find the sums of real numbers.

take note

Key Concept Adding Real Numbers

Adding Numbers With the Same Sign

To add two numbers with the same sign, add their absolute values. The sum has the same sign as the addends.

Examples $3 + 4 = 7$ $\quad -3 + (-4) = -7$

Adding Numbers With Different Signs

To add two numbers with different signs, subtract their absolute values. The sum has the same sign as the addend with the greater absolute value.

Examples $-3 + 4 = 1$ $\quad 3 + (-4) = -1$

Dynamic Activity
Adding Real Numbers

Problem 2 Adding Real Numbers

Plan

What is the first step in finding each sum? Identify whether the addends have the same sign or different signs. Then choose the appropriate rule to use.

What is each sum?

A $-12 + 7$

$-12 + 7 = -5$ The difference of the absolute values is 5. The negative addend has the greater absolute value. The sum is negative.

B $-18 + (-2)$

$-18 + (-2) = -20$ The addends have the same sign (negative), so add their absolute values. The sum is negative.

C $-4.8 + 9.5$

$-4.8 + 9.5 = 4.7$ The difference of the absolute values is 4.7. The positive addend has the greater absolute value. The sum is positive.

D $\frac{3}{4} + \left(-\frac{5}{6}\right)$

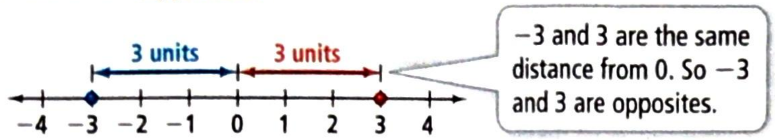
$\frac{3}{4} + \left(-\frac{5}{6}\right) = \frac{9}{12} + \left(-\frac{10}{12}\right)$ Find the least common denominator.

$= -\frac{1}{12}$ The difference of the absolute values is $\frac{1}{12}$. The negative addend has the greater absolute value. The sum is negative.

Got It? **2.** What is each sum?

a. $-16 + (-8)$ **b.** $-11 + 9$ **c.** $9 + (-11)$ **d.** $-6 + (-2)$

Two numbers that are the same distance from 0 on a number line but lie in opposite directions are **opposites.**



A number and its opposite are called **additive inverses.** To find the sum of a number and its opposite, you can use the **Inverse Property of Addition.**

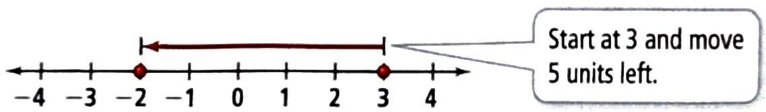
take note

Property Inverse Property of Addition

For every real number a, there is an additive inverse $-a$ such that $a + (-a) = -a + a = 0$.

Examples $14 + (-14) = 0$ $-14 + 14 = 0$

You can use opposites (additive inverses) to subtract real numbers. To see how, look at the number line below, which models $3 - 5$ and $3 + (-5)$.



$3 - 5$ and $3 + (-5)$ are equivalent expressions, illustrating the rule below.

take note

Key Concept Subtracting Real Numbers

To subtract a real number, add its opposite: $a - b = a + (-b)$.

Examples $3 - 5 = 3 + (-5) = -2$ $3 - (-5) = 3 + 5 = 8$

Problem 3 Subtracting Real Numbers

Think

Why rewrite subtraction as addition?
You can simplify expressions using the rules for adding real numbers that you learned earlier in this lesson.

What is each difference?

A $-8 - (-13) = -8 + 13$ The opposite of −13 is 13. So add 13.
$= 5$ Use rules for addition.

B $3.5 - 12.4 = 3.5 + (-12.4)$ The opposite of 12.4 is −12.4. So add −12.4.
$= -8.9$ Use rules for addition.

C $9 - 9 = 9 + (-9)$ The opposite of 9 is −9. So add −9.
$= 0$ Inverse Property of Addition

Got It? 3. a. What is $4.8 - (-8.7)$?
b. Reasoning For what values of a and b does $a - b = b - a$?

All of the addition properties of real numbers that you learned in Lesson 1-4 apply to both positive and negative numbers. You can use these properties to reorder and simplify expressions.

Problem 4 Adding and Subtracting Real Numbers

Scuba Diving **A reef explorer dives 25 ft to photograph brain coral and then rises 16 ft to travel over a ridge before diving 47 ft to survey the base of the reef. Then the diver rises 29 ft to see an underwater cavern. What is the location of the cavern in relation to sea level?**

Know	Need	Plan
Distance and direction for each change in location	Location in relation to sea level after changes	Represent the diver's trip with an expression. Reorder the values to make calculations easier.

Think

How do you represent the problem with an expression?

Start your expression with zero to represent sea level. Subtract for dives, and add for rises.

$0 - 25 + 16 - 47 + 29$	Write an expression.
$= 0 + (-25) + 16 + (-47) + 29$	Use rule for subtracting real numbers.
$= 0 + 16 + 29 + (-25) + (-47)$	Commutative Property of Addition
$= 0 + (16 + 29) + [(-25) + (-47)]$	Group addends with the same sign.
$= 0 + 45 + (-72)$	Add inside grouping symbols.
$= 45 + (-72)$	Identity Property of Addition
$= -27$	Use rule for adding numbers with different signs.

The cavern is at -27 ft in relation to sea level.

Got It? **4.** A robot submarine dives 803 ft to the ocean floor. It rises 215 ft as the water gets shallower. Then the submarine dives 2619 ft into a deep crevice. Next, it rises 734 ft to photograph a crack in the wall of the crevice. What is the location of the crack in relation to sea level?

Lesson Check

Do you know HOW?

Use a number line to find each sum.

1. $-5 + 2$

2. $-2 + (-1)$

Find each sum or difference.

3. $-12 + 9$

4. $-4 + (-3)$

5. $-3 - (-5)$

6. $1.5 - 8.5$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. Vocabulary What is the sum of a number and its opposite?

8. Compare and Contrast How is subtraction related to addition?

9. Error Analysis Your friend says that since $-a$ is the opposite of a, the opposite of a number is always negative. Describe and correct the error.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Use a number line to find each sum. See Problem 1.

10. $2 + 5$ **11.** $-3 + 8$ **12.** $4 + (-3)$ **13.** $1 + (-6)$

14. $-6 + 9$ **15.** $-4 + 7$ **16.** $-6 + (-8)$ **17.** $-9 + (-3)$

Find each sum. See Problem 2.

18. $11 + 9$ **19.** $17 + (-28)$ **20.** $12 + (-9)$ **21.** $-2 + 7$

22. $-14 + (-10)$ **23.** $-9 + (-2)$ **24.** $3.2 + 1.4$ **25.** $5.1 + (-0.7)$

26. $-2.2 + (-3.8)$ **27.** $\frac{1}{2} + \left(-\frac{7}{2}\right)$ **28.** $-\frac{2}{3} + \left(-\frac{3}{5}\right)$ **29.** $\frac{7}{9} + \left(-\frac{5}{12}\right)$

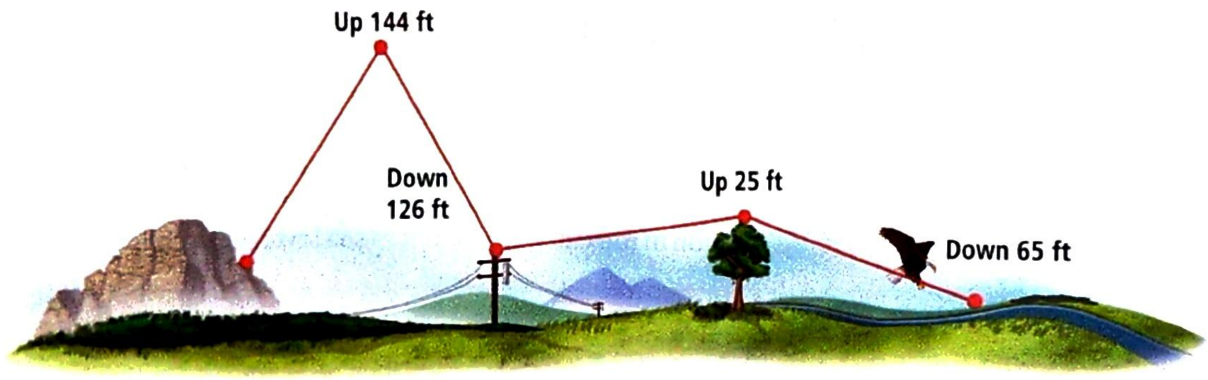
Find each difference. See Problem 3.

30. $5 - 15$ **31.** $-13 - 7$ **32.** $-19 - 7$ **33.** $36 - (-12)$

34. $-29 - (-11)$ **35.** $-7 - (-5)$ **36.** $8.5 - 7.6$ **37.** $-2.5 - 17.8$

38. $-2.9 - (-7.5)$ **39.** $3.5 - 1.9$ **40.** $\frac{1}{8} - \frac{3}{4}$ **41.** $\frac{7}{16} - \left(-\frac{1}{2}\right)$

42. Bird Watching An eagle starts flying at an elevation of 42 ft. Elevation is the distance above sea level. The diagram below shows the elevation changes during the eagle's flight. Write an expression representing the eagle's flight. What is the elevation at the brook? See Problem 4.



43. Stock Market A stock's starting price per share is \$51.47 at the beginning of the week. During the week, the price changes by gaining \$1.22, then losing \$3.47, then losing \$2.11, then losing \$.98, and finally gaining \$2.41. What is the ending stock price?

B Apply

Evaluate each expression for $a = -2$, $b = -4.1$, and $c = 5$.

44. $a - b + c$ **45.** $\cancel{c +}\ b - a$ **46.** $-a + (-c)$

47. Error Analysis Describe and correct the error in finding the difference shown at the right.

~~-4 - (-5) = -4 + (-(-5))~~
~~= -4 + 5~~
~~= -1~~

48. Writing Without calculating, tell which is greater, the sum of -135 and 257 or the sum of 135 and -257. Explain your reasoning.

Simplify each expression.

49. $1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}$ **50.** $7 + (2^2 - 3^2)$ **51.** $-2.1 - [2.3 - (3.5 - (-1.9))]$

52. Think About a Plan In golf, the expected number of strokes is called "par." When the number of strokes taken is more than par, your score is positive. When the number of strokes is less than par, your score is negative. The lowest score wins.

The scorecard shows par and one golfer's score for the first four holes played on a nine-hole golf course. The golfer's scores on the remaining five holes are $-1, 0, -1, +1, 0$. Par for the nine holes is 36. What is the golfer's total number of strokes for the nine holes?

- Can you solve the problem by adding the strokes taken on each hole?
- How is the sum of the golfer's scores related to the total number of strokes taken?

Golf Scorecard

Par	Number of Strokes	Score
4	6	+2
4	3	−1
3	3	0
5	3	−2

Reasoning **Use reasoning to determine whether the value of each expression is *positive* or *negative*. Do not calculate the exact answers.**

53. $-225 + 318$ **54.** $-\frac{7}{8} + \frac{1}{3}$ **55.** $34.5 + 12.9 - 50$

STEM **56. Temperature Scales** The Kelvin temperature scale is related to the degrees Celsius (°C) temperature scale by the formula $x = 273 + y$, where x is the number of kelvins and y is the temperature in degrees Celsius. What is each temperature in kelvins?

a. −22°C **b.** 0°C **c.** −32°C

57. Writing Explain how you can tell without calculating whether the sum of a positive number and a negative number will be positive, negative, or zero.

Decide whether each statement is true or false. Explain your reasoning.

58. The sum of a positive number and a negative number is always negative.

59. The difference of two numbers is always less than the sum of those two numbers.

60. A number minus its opposite is twice the number.

STEM **61. Meteorology** Weather forecasters use a barometer to measure air pressure and make weather predictions. Suppose a standard mercury barometer reads 29.8 in. The mercury rises 0.02 in. and then falls 0.09 in. The mercury falls again 0.18 in. before rising 0.07 in. What is the final reading on the barometer?

62. Multiple Choice Which expression is equivalent to $x - y$?

Ⓐ $y - x$ Ⓑ $x - (-y)$ Ⓒ $x + (-y)$ Ⓓ $y + (-x)$

STEM **63. Chemistry** Atoms contain particles called protons and electrons. Each proton has a charge of $+1$ and each electron has a charge of -1. A certain sulfur ion has 18 electrons and 16 protons. The charge on an ion is the sum of the charges of its protons and electrons. What is the sulfur ion's charge?

Challenge 64. **Reasoning** If $|x| > |y|$, does $|x - y| = |x| - |y|$? Justify your answer.

65. **Reasoning** A student wrote the equation $-|m| = |-m|$. Is the equation *always*, *sometimes*, or *never* true? Explain.

Simplify each expression.

66. $\frac{c}{4} - \frac{c}{4}$ **67.** $\frac{w}{5} + \left(-\frac{w}{10}\right)$ **68.** $\frac{d}{5} - \left(-\frac{d}{5}\right)$

69. **Reasoning** Answer each question. Justify your answers.

a. Is $|a - b|$ always equal to $|b - a|$? **b.** Is $|a + b|$ always equal to $|a| + |b|$?

Standardized Test Prep

SAT/ACT

70. What is the value of $-b - a$ when $a = -4$ and $b = 7$?

Ⓐ -11 Ⓑ -3 Ⓒ 3 Ⓓ 11

71. Which expression is equivalent to $19 - 41$?

Ⓕ $|19 - 41|$ Ⓖ $|19 + 41|$ Ⓗ $-|19 - 41|$ Ⓘ $-|19 + 41|$

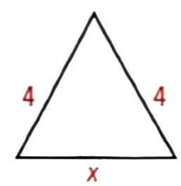
72. Which equation illustrates the Identity Property of Multiplication?

Ⓐ $x \cdot 0 = 0$ Ⓑ $x \cdot 1 = x$ Ⓒ $x(yz) = (xy)z$ Ⓓ $x \cdot y = y \cdot x$

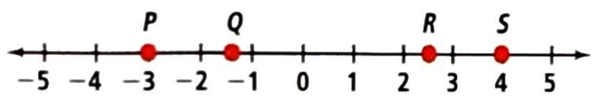
73. What is an algebraic expression for the perimeter of the triangle?

Ⓕ $8 + x$ Ⓗ 8

Ⓖ $4x$ Ⓘ $4 + x$



74. Which point on the number line below is the best estimate for $\sqrt{8}$?



Ⓐ P Ⓑ Q Ⓒ R Ⓓ S

Mixed Review

Tell whether the expressions in each pair are equivalent. See Lesson 1-4.

75. $\frac{3}{4} \cdot d \cdot 4$ and $3d$ **76.** $(2.1 \cdot h) \cdot 3$ and $6.3 + h$ **77.** $(6 + b) + a$ and $6 + (a + b)$

Name the subset(s) of real numbers to which each number belongs. See Lesson 1-3.

78. $\frac{1}{3}$ **79.** -5.333 **80.** $\sqrt{16}$ **81.** 82.0371 **82.** $\sqrt{21}$

Get Ready! **To prepare for Lesson 1-6, do Exercises 83–85.**

Evaluate each expression for $a = 2$, $h = 5$, and $w = 8$. See Lesson 1-2.

83. $4h - 5a \div w$ **84.** $a^2w - h^2 + 2h$ **85.** $(w^2h - a^2) + 12 \div 3a$

Concept Byte

Use With Lesson 1-5

Always, Sometimes, or Never

Content Standard

Prepares for A.CED.3 Represent constraints by equations or inequalities . . .

A statement can be always, sometimes, or never true. For each activity, work with a group of 4 students. Take turns predicting each answer. If the predictor gives a correct answer and explanation, he or she scores 1 point. Otherwise, the first person who proves the predictor incorrect scores 1 point. Whoever has the most points at the end of an activity wins.

Activity 1

Is each description *always, sometimes,* or *never* true about the members of your group?

1. takes an algebra class

2. lives in your state

3. plays a musical instrument

4. is less than 25 years old

5. speaks more than one language

6. is taller than 5 m

7. has a sibling

8. plays basketball

Activity 2

Suppose each member of your group takes one of the four cards at the right. Will a group member *always, sometimes,* or *never* have a number that fits each description?

9. greater than 2

10. greater than 25

11. even

12. irrational number

13. prime number

14. rational number

15. divisible by 2

16. less than 10

Activity 3

Each member of your group substitutes any integer for x in each statement. Will a group member *always, sometimes,* or *never* have a true statement?

17. $x - 2$ is greater than x.

18. $|x|$ is less than x.

19. $7 + x = x + 7$

20. $13 - x = x - 13$

21. $x + 0 = x$

22. $-4 + (3 + x) = x + (-4 + 3)$

23. $x \cdot 0 = 0$

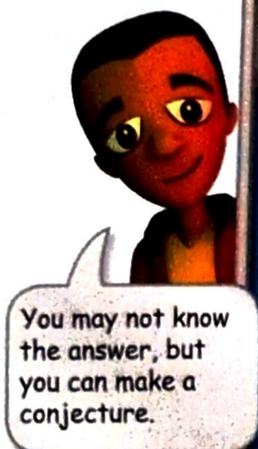
24. $|x|$ is greater than x.

1-6 Multiplying and Dividing Real Numbers

Content Standard

Prepares for N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

Objective To find products and quotients of real numbers



MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

Use patterns to complete the table and answer the questions below. Explain your reasoning.

- What is the sign of the product of a positive number and a negative number?
- What is the sign of the product of two negative numbers?

$2 \cdot 3 = 6$	$-2 \cdot 3 = -6$
$2 \cdot 2 = 4$	$-2 \cdot 2 = -4$
$2 \cdot 1 = 2$	$-2 \cdot 1 = -2$
$2 \cdot 0 = \blacksquare$	$-2 \cdot 0 = \blacksquare$
$2 \cdot (-1) = \blacksquare$	$-2 \cdot (-1) = \blacksquare$
$2 \cdot (-2) = \blacksquare$	$-2 \cdot (-2) = \blacksquare$

Lesson Vocabulary
- multiplicative inverse
- reciprocal

The patterns in the Solve It suggest rules for multiplying real numbers.

Essential Understanding The rules for multiplying real numbers are related to the properties of real numbers and the definitions of operations.

You know that the product of two positive numbers is positive. For example, $3(5) = 15$. You can think about the product of a positive number and a negative number in terms of groups of numbers. For example, $3(-5)$ means 3 groups of -5. So, $3(-5) = (-5) + (-5) + (-5)$, or $3(-5) = -15$.

You can also derive the product of two negative numbers, such as $-3(-5)$.

$3(-5) = -15$	Start with the product $3(-5) = -15$.
$-[3(-5)] = -(-15)$	The opposites of two equal numbers are equal.
$-1[3(-5)] = -(-15)$	Multiplication Property of -1
$[-1(3)](-5) = -(-15)$	Associative Property of Multiplication
$-3(-5) = -(-15)$	Multiplication Property of -1
$-3(-5) = 15$	The opposite of -15 is 15.

These discussions illustrate the following rules for multiplying real numbers.

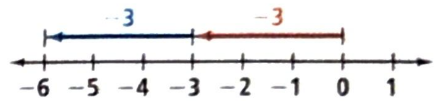
take note

Key Concept Multiplying Real Numbers

Words The product of two real numbers with *different* signs is *negative.*

Examples $2(-3) = -6$ $\quad -2 \cdot 3 = -6$

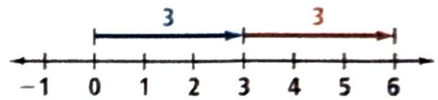
Model $2(-3) = -6$



Words The product of two real numbers with the *same* sign is *positive.*

Examples $2 \cdot 3 = 6$ $\quad -2(-3) = 6$

Model $2 \cdot 3 = 6$



Problem 1 Multiplying Real Numbers

Plan

What is your first step in finding a product of real numbers?
Identify the signs of the factors. Then determine the sign of the product.

What is each product?

A $12(-8) = -96$ The product of two numbers with different signs is negative.

B $24(0.5) = 12$ The product of two numbers with the same sign is positive.

C $-\frac{3}{4} \cdot \frac{1}{2} = -\frac{3}{8}$ The product of two numbers with different signs is negative.

D $(-3)^2 = (-3)(-3) = 9$ The product of two numbers with the same sign is positive.

Got It? **1.** What is each product?

a. $6(-15)$ **b.** $12(0.2)$ **c.** $-\frac{7}{10}\left(\frac{3}{5}\right)$ **d.** $(-4)^2$

Notice that $(-3)^2 = 9$ in part (d) of Problem 1. Recall from Lesson 1-3 that a is a square root of b if $a^2 = b$. So, -3 is a square root of 9. A negative square root is represented by $-\sqrt{}$. Every positive real number has a positive and a negative square root. The symbol $\pm$ in front of the radical indicates both square roots.

Problem 2 Simplifying Square Root Expressions

Think

How can you find a negative square root?
Look for a negative number that you can multiply by itself to get a product that is equal to the radicand.

What is the simplified form of each expression?

A $-\sqrt{25} = -5$ $\quad (-5)^2 = 25$, so $-\sqrt{25} = -5$.

B $\pm\sqrt{\frac{4}{49}} = \pm\frac{2}{7}$ $\quad \left(\frac{2}{7}\right)^2 = \frac{4}{49}$ and $\left(-\frac{2}{7}\right)^2 = \frac{4}{49}$, so $\pm\sqrt{\frac{4}{49}} = \pm\frac{2}{7}$.

Got It? **2.** What is the simplified form of each expression?

a. $\sqrt{64}$ **b.** $\pm\sqrt{16}$ **c.** $-\sqrt{121}$ **d.** $\pm\sqrt{\frac{1}{36}}$

Essential Understanding Rules for dividing real numbers are related to the rules for multiplying real numbers.

For any real numbers a, b, and c where $a \neq 0$, if $a \cdot b = c$, then $b = c \div a$. For instance, $-8(-2) = 16$, so $-2 = 16 \div (-8)$. Similarly $-8(2) = -16$, so $2 = -16 \div (-8)$. These examples illustrate the following rules.

take note

Key Concept Dividing Real Numbers

Words The quotient of two real numbers with *different* signs is *negative*.

Examples $-20 \div 5 = -4$ $\quad$ $20 \div (-5) = -4$

Words The quotient of two real numbers with the *same* sign is *positive*.

Examples $20 \div 5 = 4$ $\quad$ $-20 \div (-5) = 4$

Division Involving 0

Words The quotient of 0 and any nonzero real number is 0. The quotient of any real number and 0 is undefined.

Examples $0 \div 8 = 0$ $\quad$ $8 \div 0$ is undefined.

Problem 3 Dividing Real Numbers

Think
How is dividing similar to multiplying?
You find the sign of a quotient using the signs of the numbers you're dividing, just as you find the sign of a product using the signs of the factors.

Sky Diving **A sky diver's elevation changes by −3600 ft in 4 min after the parachute opens. What is the average change in the sky diver's elevation each minute?**

$-3600 \div 4 = -900$ $\quad$ The numbers have different signs, so the quotient is negative.

The sky diver's average change in elevation is −900 ft per minute.

Got It? **3.** You make five withdrawals of equal amounts from your bank account. The total amount you withdraw is \$360. What is the change in your account balance each time you make a withdrawal?

The Inverse Property of Multiplication describes the relationship between a number and its multiplicative inverse.

take note

Property Inverse Property of Multiplication

Words For every nonzero real number a, there is a **multiplicative inverse** $\frac{1}{a}$ such that $a\left(\frac{1}{a}\right) = 1$.

Examples The multiplicative inverse of -4 is $-\frac{1}{4}$ because $-4\left(-\frac{1}{4}\right) = 1$.

The **reciprocal** of a nonzero real number of the form $\frac{a}{b}$ is $\frac{b}{a}$. The product of a number and its reciprocal is 1, so the reciprocal of a number is its multiplicative inverse. This suggests a rule for dividing fractions.

Here's Why It Works Let a, b, c, and d be nonzero integers.

$\frac{a}{b} \div \frac{c}{d} = \dfrac{\frac{a}{b}}{\frac{c}{d}}$	Write the expression as a fraction.
$= \dfrac{\frac{a}{b} \cdot \frac{d}{c}}{\frac{c}{d} \cdot \frac{d}{c}}$	Multiply the numerator and denominator by $\frac{d}{c}$. Since this is equivalent to multiplying by 1, it does not change the quotient.
$= \dfrac{\frac{a}{b} \cdot \frac{d}{c}}{1}$	Inverse Property of Multiplication
$= \frac{a}{b} \cdot \frac{d}{c}$	Simplify.

This shows that dividing by a fraction is equivalent to multiplying by the reciprocal of the fraction.

Problem 4 Dividing Fractions

Multiple Choice What is the value of $\frac{x}{y}$ when $x = -\frac{3}{4}$ and $y = -\frac{2}{3}$?

Ⓐ $-\frac{9}{8}$ Ⓑ $-\frac{1}{2}$ Ⓒ $\frac{1}{2}$ Ⓓ $\frac{9}{8}$

Think	Write
Rewrite the expression.	$\frac{x}{y} = x \div y$
Substitute $-\frac{3}{4}$ for x and $-\frac{2}{3}$ for y.	$= -\frac{3}{4} \div \left(-\frac{2}{3}\right)$
Multiply by the reciprocal of $-\frac{2}{3}$.	$= -\frac{3}{4} \cdot \left(-\frac{3}{2}\right)$
Simplify. Since both factors are negative, the product is positive.	$= \frac{9}{8}$

The correct answer is D.

Got It? **4. a.** What is the value of $\frac{3}{4} \div \left(-\frac{5}{2}\right)$?

b. Reasoning Is $\frac{3}{4} \div \left(-\frac{5}{2}\right)$ equivalent to $-\left(\frac{3}{4} \div \frac{5}{2}\right)$? Explain.

Lesson Check

Do you know HOW?

Find each product. Simplify, if necessary.

1. $-3(-12)$ **2.** $\frac{5}{8}\left(-\frac{2}{8}\right)$

Find each quotient. Simplify, if necessary.

3. $-48 \div 3$ **4.** $-\frac{9}{10} \div \left(-\frac{4}{5}\right)$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary What is the reciprocal of $-\frac{1}{5}$?

6. Reasoning Use a number line to explain why $-15 \div 3 = -5$.

7. Reasoning Determine how many real square roots each number has. Explain your answers.

a. 49 **b.** 0

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find each product. Simplify, if necessary. See Problem 1.

8. $-8(12)$ **9.** $8(12)$ **10.** $7(-9)$ **11.** $5 \cdot 4.1$

12. $-7 \cdot 1.1$ **13.** $10(-2.5)$ **14.** $6\left(-\frac{1}{4}\right)$ **15.** $-\frac{1}{9}\left(-\frac{3}{4}\right)$

16. $-\frac{3}{7} \cdot \frac{9}{10}$ **17.** $-\frac{2}{11}\left(-\frac{11}{2}\right)$ **18.** $\left(-\frac{2}{9}\right)^2$ **19.** $(-1.2)^2$

Simplify each expression. See Problem 2.

20. $\sqrt{400}$ **21.** $\sqrt{169}$ **22.** $-\sqrt{16}$ **23.** $-\sqrt{900}$ **24.** $\sqrt{\frac{36}{49}}$

25. $-\sqrt{\frac{25}{81}}$ **26.** $-\sqrt{\frac{1}{9}}$ **27.** $-\sqrt{\frac{121}{16}}$ **28.** $\pm\sqrt{1.96}$ **29.** $\pm\sqrt{0.25}$

Find each quotient. Simplify, if necessary. See Problem 3.

30. $48 \div 3$ **31.** $-84 \div 14$ **32.** $-39 \div (-13)$ **33.** $\frac{63}{-21}$

34. $-46 \div (-2)$ **35.** $-8.1 \div 9$ **36.** $\frac{-121}{11}$ **37.** $75 \div (-0.3)$

STEM **38. Scuba Diving** A scuba diver's vertical position in relation to the surface of the water changes by -90 ft in 3 min. What is the average change in the diver's vertical position each minute?

39. Part-Time Job You earn the same amount each week at your part-time job. The total amount you earn in 4 weeks is \$460. How much do you earn per week?

Find each quotient. Simplify, if necessary. See Problem 4.

40. $20 \div \frac{1}{4}$ **41.** $-5 \div \left(-\frac{5}{3}\right)$ **42.** $\frac{9}{10} \div \left(-\frac{4}{5}\right)$ **43.** $-\frac{12}{13} \div \frac{12}{13}$

Find the value of the expression $\frac{x}{y}$ for the given values of x and y. Write your answer in the simplest form.

44. $x = -\frac{2}{3}; y = -\frac{1}{4}$ **45.** $x = -\frac{5}{6}; y = \frac{3}{5}$ **46.** $x = \frac{2}{7}; y = -\frac{20}{21}$ **47.** $x = \frac{3}{8}; y = \frac{3}{4}$

B Apply

48. Think About a Plan A lumberjack cuts 7 pieces of equal length from a log, as shown at the right. What is the change in the log's length after 7 cuts?

- What operation can you use to find the answer?
- Will your answer be a positive value or a negative value? How do you know?

49. Farmer's Market A farmer has 120 bushels of beans for sale at a farmer's market. He sells an average of $15\frac{3}{4}$ bushels each day. After 6 days, what is the change in the total number of bushels the farmer has for sale at the farmer's market?

50. Stocks The price per share of a stock changed by $-\$4.50$ on each of 5 consecutive days. If the starting price per share was \$67.50, what was the ending price?

Open-Ended **Write an algebraic expression that uses x, y, and z and simplifies to the given value when $x = -3$, $y = -2$, and $z = -1$. The expression should involve only multiplication or division.**

51. -16 **52.** 1 **53.** 12

Evaluate each expression for $m = -5$, $n = \frac{3}{2}$, and $p = -8$.

54. $-7m - 10n$

55. $-3mnp$

56. $8n \div (-6p)$

57. $2p^2(-n) \div m$

58. Look for a Pattern Extend the pattern in the diagram to six factors of -2. What rule describes the sign of the product based on the number of negative factors?

$-2(-2) = 4$
$-2(-2)(-2) = -8$
$-2(-2)(-2)(-2) = 16$

STEM **59. Temperature** The formula $F = \frac{9}{5}C + 32$ changes a temperature reading from the Celsius scale C to the Fahrenheit scale F. What is the temperature measured in degrees Fahrenheit when the Celsius temperature is -25°C?

60. Reasoning Suppose a and b are integers. Describe what values of a and b make the statement true.

a. Quotient $\frac{a}{b}$ is positive.

b. Quotient $\frac{a}{b}$ is negative.

c. Quotient $\frac{a}{b}$ is equal to 0.

d. Quotient $\frac{a}{b}$ is undefined.

61. Writing Explain how to find the quotient of $-1\frac{2}{3}$ and $-2\frac{1}{2}$.

62. Reasoning Do you think a negative number raised to an even power will be positive or negative? Explain.

63. History The Rhind Papyrus is one of the best-known examples of Egyptian mathematics. One problem solved on the Rhind Papyrus is $100 \div 7\frac{7}{8}$. What is the solution of this problem?

64. **Error Analysis** Describe and correct the error in dividing the fractions at the right.

$$-\frac{3}{4} \div \frac{2}{5} = -\frac{4}{3} \cdot \frac{2}{5} = -\frac{8}{15}$$

65. **Reasoning** You can derive the rule for division involving 0 shown on page 40.
   - **a.** Suppose $0 \div x = y$, where $x \neq 0$. Show that $y = 0$. (*Hint*: If $0 \div x = y$, then $x \cdot y = 0$ by the definition of division.)
   - **b.** If $x \neq 0$, show that there is no value of y such that $x \div 0 = y$. (*Hint*: Suppose there is a value of y such that $x \div 0 = y$. What would this imply about x?)

Challenge **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.**

66. The product of a number and its reciprocal is -1.

67. The quotient of a nonzero number and its opposite is -1.

68. If the product of two fractions is negative, then their quotient is positive.

69. **Reasoning** What is the greatest integer n for which $(-n)^3$ is positive and the value of the expression has a 2 in the ones place?

Standardized Test Prep

SAT/ACT

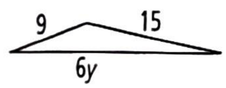
70. Which expression does NOT have the same value as $-11 + (-11) + (-11)$?

(A) -33 (B) $3(-11)$ (C) $(-11)^3$ (D) $33 - 66$

71. Miguel measured the area of a piece of carpet and figured out that the approximate error was $3|-0.2|$. What is the decimal form of $3|-0.2|$?

(F) -0.6 (G) -0.06 (H) 0.06 (I) 0.6

72. What is the perimeter of the triangle shown?



(A) $6y + 24$ (C) $15y + 15$

(B) $21y + 9$ (D) $30y$

Mixed Review

Find each difference. See Lesson 1-5.

73. $46 - 16$ 74. $34 - 44$ 75. $-37 - (-27)$

Get Ready! **To prepare for Lesson 1-7, do Exercises 76–78.**

Name the property that each statement illustrates. See Lesson 1-4.

76. $-x + 0 = -x$ 77. $13(-11) = -11(13)$ 78. $-5 \cdot (m \cdot 8) = (-5 \cdot m) \cdot 8$

Concept Byte

Use With Lesson 1-6

Operations With Rational and Irrational Numbers

Content Standard

N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

For each sum, determine whether the result is a rational number or an irrational number.

1. $\frac{5}{8} + \frac{3}{5}$

2. $-\frac{1}{4} + \frac{2}{5}$

3. $-\frac{1}{2} + \sqrt{2}$

4. $\sqrt{5} + \left(-\frac{3}{11}\right)$

5. $\frac{1}{4} + \sqrt{12}$

6. $-\frac{3}{11} + \left(-\frac{1}{3}\right)$

For each product, determine whether the result is a rational number or an irrational number.

7. $\frac{1}{2} \cdot \frac{2}{15}$

8. $\sqrt{2} \cdot \frac{2}{5}$

9. $-\frac{3}{5} \cdot \frac{4}{9}$

10. $\frac{5}{8} \cdot \sqrt{7}$

11. $-\frac{3}{4} \cdot \frac{2}{9}$

12. $-\frac{4}{9} \cdot -\sqrt{5}$

For Exercises 13–16, predict whether the sum or product will be a rational or irrational number. Explain.

13. The sum of two rational numbers.

14. The product of a nonzero rational number and an irrational number.

15. The product of two rational numbers.

16. The sum of a rational number and an irrational number.

17. Can the sum of two irrational numbers be rational? If so, give an example. If not, explain why not.

18. Can the product of two irrational numbers be rational? If so, give an example. If not, explain why not.

1-7 The Distributive Property

Content Standard

A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.

Objective To use the Distributive Property to simplify expressions

There's more than one way to figure this out.

MATHEMATICAL PRACTICES

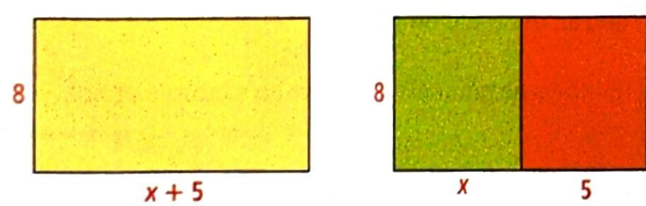
SOLVE IT! Getting Ready!

In your favorite video game, you rotate shapes as they fall to make them fit together in a rectangle. When you complete an entire row, you score 1 point for each square in that row.

The screen at the right shows your latest game in pause mode. Using only the shapes shown, what is the maximum possible score for this game? Explain your reasoning.

Lesson Vocabulary
- Distributive Property
- term
- constant
- coefficient
- like terms

To solve problems in mathematics, it is often useful to rewrite expressions in simpler forms. The **Distributive Property,** illustrated by the area model below, is another property of real numbers that helps you to simplify expressions.



The model shows that $8(x + 5) = 8(x) + 8(5)$.

Essential Understanding You can use the Distributive Property to simplify the product of a number and a sum or difference.

take note

Property Distributive Property

Let a, b, and c be real numbers.

Algebra	Examples
$a(b + c) = ab + ac$	$4(20 + 6) = 4(20) + 4(6)$
$(b + c)a = ba + ca$	$(20 + 6)4 = 20(4) + 6(4)$
$a(b - c) = ab - ac$	$7(30 - 2) = 7(30) - 7(2)$
$(b - c)a = ba - ca$	$(30 - 2)7 = 30(7) - 2(7)$

Problem 1 Simplifying Expressions

Think
How do you read expressions like $3(x+8)$?
Read an expression inside parentheses as "the quantity." Read $3(x+8)$ as "3 times the quantity x plus 8."

What is the simplified form of each expression?

A $3(x+8)$

$3(x+8) = 3(x) + 3(8)$ Distributive Property

$= 3x + 24$ Simplify.

B $(5b-4)(-7)$

$(5b-4)(-7) = 5b(-7) - 4(-7)$

$= -35b + 28$

Got It? 1. What is the simplified form of each expression?

a. $5(x+7)$ **b.** $12\left(3 - \frac{1}{6}t\right)$ **c.** $(0.4 + 1.1c)3$ **d.** $(2y-1)(-y)$

Recall that a fraction bar may act as a grouping symbol. A fraction bar indicates division. Any fraction $\frac{a}{b}$ can also be written as $a \cdot \frac{1}{b}$. You can use this fact and the Distributive Property to rewrite some fractions as sums or differences.

Problem 2 Rewriting Fraction Expressions

Think
How can you get started?
Think of division as multiplying by the reciprocal. So change the division by 5 to multiplication by $\frac{1}{5}$.

What sum or difference is equivalent to $\frac{7x+2}{5}$?

$\frac{7x+2}{5} = \frac{1}{5}(7x+2)$ Write division as multiplication.

$= \frac{1}{5}(7x) + \frac{1}{5}(2)$ Distributive Property

$= \frac{7}{5}x + \frac{2}{5}$ Simplify.

Got It? 2. What sum or difference is equivalent to each expression?

a. $\frac{4x-16}{3}$ **b.** $\frac{11+3x}{6}$ **c.** $\frac{15+6x}{12}$ **d.** $\frac{4-2x}{8}$

The Multiplication Property of -1 states that $-1 \cdot x = -x$. To simplify an expression such as $-(x+6)$, you can rewrite the expression as $-1(x+6)$.

Problem 3 Using the Multiplication Property of -1

Think
What does the negative sign in front of the parentheses mean?
It indicates the opposite of the entire expression inside the parentheses.

Multiple Choice What is the simplified form of $-(2y-3x)$?

Ⓐ $2y + 3x$ Ⓑ $-2y + (-3x)$ Ⓒ $-2y + 3x$ Ⓓ $2y - 3x$

$-(2y-3x) = -1(2y-3x)$ Multiplication Property of -1

$= (-1)(2y) + (-1)(-3x)$ Distributive Property

$= -2y + 3x$ Simplify.

The correct choice is C.

Got It? 3. What is the simplified form of each expression?

a. $-(a+5)$ **b.** $-(-x+31)$ **c.** $-(4x-12)$ **d.** $-(6m-9n)$

You can use the Distributive Property to make calculations easier to do with mental math. Some numbers can be thought of as simple sums or differences.

Problem 4 Using the Distributive Property for Mental Math

Eating Out **Deli sandwiches cost $4.95 each. What is the total cost of 8 sandwiches? Use mental math.**

Know	Need	Plan
• Sandwiches cost $4.95. • You are buying 8 sandwiches.	Total cost of 8 sandwiches	Express $4.95 as a difference and use the Distributive Property.

Think

How can you express decimals as simple sums and differences?

Think of a decimal as the sum or difference of its whole number portion and its decimal portion.

The total cost is the product of the number of sandwiches you buy, 8, and the cost per sandwich, $4.95.

$8(4.95) = 8(5 - 0.05)$		Think of 4.95 as $5 - 0.05$.
$= 8(5) - 8(0.05)$		Distributive Property
$= 40 - 0.4$		Multiply mentally.
$= 39.6$		Subtract mentally.

The total cost for 8 sandwiches is $39.60.

Got It? **4.** Julia commutes to work on the train 4 times each week. A round-trip ticket costs $7.25. What is her weekly cost for tickets? Use mental math.

Essential Understanding You can simplify an algebraic expression by combining the parts of the expression that are alike.

In an algebraic expression, a **term** is a number, a variable, or the product of a number and one or more variables. A **constant** is a term that has no variable. A **coefficient** is a numerical factor of a term. Rewrite expressions as sums to identify these parts of an expression.

$6a^2$, $-5ab$, $3b$, and -12 are terms.

$$6a^2 - 5ab + 3b - 12 = 6a^2 + (-5ab) + 3b + (-12)$$

coefficients constant

In the algebraic expression $6a^2 - 5ab + 3b - 12$, the terms have coefficients of 6, -5, and 3. The term -12 is a constant.

Like terms have the same variable factors. To identify like terms, compare the variable factors of the terms, as shown below.

Terms	$7a$ and $-3a$	$4x^2$ and $12x^2$	$6ab$ and $-2a$	xy^2 and x^2y
Variable Factors	a and a	x^2 and x^2	ab and a	xy^2 and x^2y
Like Terms?	yes	yes	no	no

An algebraic expression in simplest form has no like terms or parentheses.

Not Simplified	Simplified
$2(3x - 5 + 4x)$	$14x - 10$

You can use the Distributive Property to help combine like terms. Think of the Distributive Property as $ba + ca = (b + c)a$.

Problem 5 Combining Like Terms

Plan

What terms can you combine?
You can combine any terms that have exactly the same variables with exactly the same exponents.

What is the simplified form of each expression?

A $8x^2 + 2x^2$

$8x^2 + 2x^2 = (8 + 2)x^2$ Distributive Property

$= 10x^2$ Simplify.

B $5x - 3 - 3x + 6y + 4$

$5x - 3 - 3x + 6y + 4 = 5x + (-3) + (-3x) + 6y + 4$ Rewrite as a sum.

$= 5x + (-3x) + 6y + (-3) + 4$ Commutative Property

$= (5 - 3)x + 6y + (-3) + 4$ Distributive Property

$= 2x + 6y + 1$ Simplify.

Got It? **5.** What is the simplified form of each expression in parts (a)–(c)?

a. $3y - y$ **b.** $-7mn^4 - 5mn^4$ **c.** $7y^3z - 6yz^3 + y^3z$

d. Reasoning Can you simplify $8x^2 - 2x^4 - 2x + 2 + xy$ further? Explain.

Lesson Check

Do you know HOW?

1. What is the simplified form of each expression? Use the Distributive Property.

a. $(j + 2)7$

b. $-8(x - 3)$

c. $-(4 - c)$

d. $-(11 + 2b)$

Rewrite each expression as a sum.

2. $-8x^2 + 3xy - 9x - 3$

3. $2ab - 5ab^2 - 9a^2b$

Tell whether the terms are like terms.

4. $3a$ and $-5a$ **5.** $2xy^2$ and $-x^2y$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. Vocabulary Does each equation demonstrate the Distributive Property? Explain.

a. $-2(x + 1) = -2x - 2$

b. $(s - 4)8 = 8(s - 4)$

c. $5n - 45 = 5(n - 9)$

d. $8 + (t + 6) = (8 + t) + 6$

7. Mental Math How can you express 499 to find the product 499×5 using mental math? Explain.

8. Reasoning Is each expression in simplified form? Justify your answer.

a. $4xy^3 + 5x^3y$

b. $-(y - 1)$

c. $5x^2 + 12xy - 3yx$

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Use the Distributive Property to simplify each expression.

See Problem 1.

9. $6(a + 10)$
10. $8(4 + x)$
11. $(5 + w)5$
12. $(2t + 3)11$
13. $10(9 - t)$
14. $12(2j - 6)$
15. $16(7b + 6)$
16. $(1 + 3d)9$
17. $(3 - 8c)1.5$
18. $(5w - 15)2.1$
19. $\frac{1}{4}(4f - 8)$
20. $6\left(\frac{1}{3}h + 1\right)$
21. $(-8z - 10)(-1.5)$
22. $0(3.7x - 4.21)$
23. $1\left(\frac{3}{11} - \frac{7d}{17}\right)$
24. $\frac{1}{2}\left(\frac{1}{2}y - \frac{1}{2}\right)$

Write each fraction as a sum or difference.

See Problem 2.

25. $\frac{2x + 7}{5}$
26. $\frac{17 + 5n}{4}$
27. $\frac{8 - 9x}{3}$
28. $\frac{4y - 12}{2}$
29. $\frac{25 - 8t}{5}$
30. $\frac{18x + 51}{17}$
31. $\frac{22 - 2n}{2}$
32. $\frac{42w + 14}{7}$

Simplify each expression.

See Problem 3.

33. $-(20 + d)$
34. $-(-5 - 4y)$
35. $-(9 - 7c)$
36. $-(-x + 15)$
37. $-(18a - 17b)$
38. $-(2.1c - 4d)$
39. $-(-m + n + 1)$
40. $-(x + 3y - 3)$

Use mental math to find each product.

See Problem 4.

41. 5.1×8
42. 3×7.25
43. 299×3
44. 4×197
45. 3.9×6
46. 5×2.7
47. 6.15×4
48. 6×9.1

49. You buy 50 of your favorite songs from a Web site that charges $.99 for each song. What is the cost of 50 songs? Use mental math.

50. The perimeter of a baseball diamond is about 360 ft. If you take 12 laps around the diamond, what is the total distance you run? Use mental math.

51. One hundred and five students see a play. Each ticket costs $45. What is the total amount the students spend for tickets? Use mental math.

52. Suppose the distance you travel to school is 5 mi. What is the total distance for 197 trips from home to school? Use mental math.

Simplify each expression by combining like terms.

See Problem 5.

53. $11x + 9x$
54. $8y - 7y$
55. $5t - 7t$
56. $-n + 4n$
57. $5w^2 + 12w^2$
58. $2x^2 - 9x^2$
59. $-4y^2 + 9y^2$
60. $6c - 4 + 2c - 7$
61. $5 - 3x + y + 6$
62. $2n + 1 - 4m - n$
63. $-7h + 3h^2 - 4h - 3$
64. $10ab + 2ab^2 - 9ab$

B Apply

Write a word phrase for each expression. Then simplify each expression.

65. $3(t - 1)$
66. $4(d + 7)$
67. $\frac{1}{3}(6x - 1)$

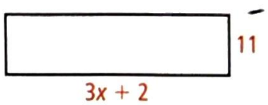
STEM 68. Physiology The recommended heart rate for exercise, in beats per minute, is given by the expression $0.8(200 - y)$ where y is a person's age in years. Rewrite this expression using the Distributive Property. What is the recommended heart rate for a 20-year-old person? For a 50-year-old person? Use mental math.

69. Error Analysis Identify and correct the error shown at the right.

$$\cancel{4(x+5) = (4 \cdot x)(4 \cdot 5) = 80x}$$

70. Error Analysis A friend uses the Distributive Property to simplify $4(2b - 5)$ and gets $8b - 5$ as the result. Describe and correct the error.

Geometry **Write an expression in simplified form for the area of each rectangle.**

71.


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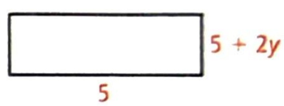

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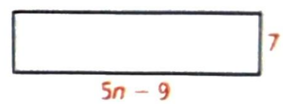


74. Think About a Plan You are replacing your regular shower head with a water-saving shower head. These shower heads use the amount of water per minute shown. If you take an 8-min shower, how many gallons of water will you save?

- Which would you use to represent water saved each minute, an expression involving addition or an expression involving subtraction?
- How can you use the Distributive Property to find the total amount of water saved?

Simplify each expression.

75. $6yz + 2yz - 8yz$

76. $-2ab + ab + 9ab - 3ab$

77. $-9m^3n + 4m^3n + 5mn$

78. $3(-4cd - 5)$

79. $12x^2y - 8x^2y^2 + 11x^2y - 4x^3y^2 - 9xy^2$

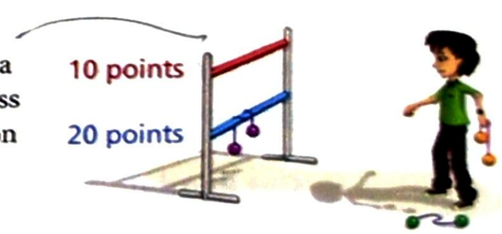
80. $a - \frac{a}{4} + \frac{3}{4}a$

81. Reasoning The Distributive Property also applies to division, as shown.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

Use the Distributive Property of Division to rewrite $\frac{9 + 12n}{3}$. Then simplify.

82. Lawn Game You play a game where you throw a pair of connected balls at a structure, as shown at the right. When a pair wraps around a bar, you earn the points shown. You toss 3 pairs, and all of them wrap around a bar. Which expression could represent your total score if a pairs of balls wrap around the blue bar?



Ⓐ $30 + 10a$

Ⓑ $20a + 3 - 10a$

Ⓒ $10a + 20(3 - a)$

Ⓓ $30a + 10$

83. **Open-Ended** Suppose you used the Distributive Property to get the expression $3m - 6n - 15$. With what expression could you have started?

Challenge

84. **Writing** Your friend uses the order of operations to find the value of $11(39 - 3)$. Would you prefer to use the Distributive Property instead? Explain.

Simplify each expression.

85. $5(2d + 1) + 7(5d + 3)$
86. $6(4t - 3) + 6(4 - 3t)$
87. $9(5 + t) - 7(t + 3)$
88. $4(r + 8) - 5(2r - 1)$
89. $-(m + 9n - 12)$
90. $-6(3 - 3x - 7y) + 2y - x$

Standardized Test Prep

SAT/ACT

91. What is the simplified form of the expression $2(7c - 1)$?

A $14c - 1$ B $9c - 3$ C $14c - 2$ D $9c - 1$

92. You have already traveled 2.3 mi in a canoe. You continue to travel 0.1 mi each minute. The expression $0.1m + 2.3$ gives the distance traveled (in miles) after m minutes. What is your distance traveled after 25 min?

F 2.5 mi G 2.55 mi H 4.8 mi I 27.3 mi

93. The table at the right shows the depth several submersible vehicles can reach. Which of the submersibles are capable of diving to 12,500 ft?

A *Clelia* and *Pisces* V
B *Alvin*, *Clelia*, and *Pisces* V
C *Alvin* and *Mir* I
D *Mir* I

Depth of Submersibles

Submersible	Depth (ft)
Alvin	14,764
Clelia	1000
Mir I	20,000
Pisces V	6280

SOURCE: National Oceanic and Atmospheric Administration

94. Which expression gives the value in dollars of n nickels?

F $0.05n$ G $0.05 + n$ H $0.5n$ I $5n$

Mixed Review

Find each product.

See Lesson 1-6.

95. -5^2
96. $\left(-\frac{3}{4}\right)^2$
97. $(-1.2)^2$

Get Ready! **To prepare for Lesson 1-8, do Exercises 98–100.**

Write a word phrase for each algebraic expression.

See Lesson 1-1.

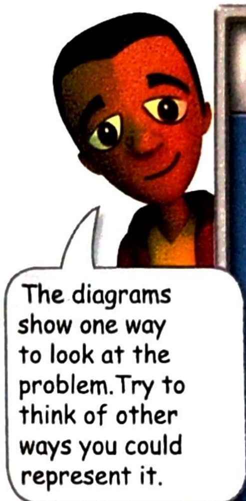
98. $x - 10$
99. $5x - 18$
100. $\frac{7}{y} + 12$

1-8 An Introduction to Equations

Content Standard

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Objective To solve equations using tables and mental math



MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

An animal shelter has a fundraising goal of $7500. The diagram shows the results for the first three weeks. The first amount is negative due to upfront costs. When will the shelter reach its goal? Make an estimate. State any assumptions and justify your reasoning.

Dynamic Activity Using Algebraic Equations

Lesson Vocabulary
- equation
- open sentence
- solution of an equation

The problem in the Solve It can be modeled by an equation. An **equation** is a mathematical sentence that uses an equal sign $(=)$.

Essential Understanding You can use an equation to represent the relationship between two quantities that have the same value.

An equation is true if the expressions on either side of the equal sign are equal $(1 + 1 = 2, x + x = 2x)$. An equation is false if the expressions on either side of the equal sign are not equal $(1 + 1 = 3, x + x = 3x)$. An equation is an **open sentence** if it contains one or more variables and may be true or false depending on the values of its variables.

Problem 1 Classifying Equations

Plan

How do you classify an equation?

If an equation contains only numbers, simplify the expressions on either side to determine if they are equal. If there is a variable in the equation, it is open.

Is the equation *true*, *false*, or *open*? Explain.

A $24 + 18 = 20 + 22$ True, because both expressions equal 42

B $7 \cdot 8 = 54$ False, because $7 \cdot 8 = 56$ and $56 \neq 54$

C $2x - 14 = 54$ Open, because there is a variable

Got It? 1. Is the equation *true*, *false*, or *open*? Explain.

a. $3y + 6 = 5y - 8$ **b.** $16 - 7 = 4 + 5$ **c.** $32 \div 8 = 2 \cdot 3$

A **solution of an equation** containing a variable is a value of the variable that makes the equation true.

Problem 2 Identifying Solutions of an Equation

Plan

How can you tell if a number is a solution of an equation?
Substitute the number for the variable in the equation. Simplify each side to see if you get a true statement.

Is $x = 6$ a solution of the equation $32 = 2x + 12$?

$32 = 2x + 12$

$32 \stackrel{?}{=} 2(6) + 12$ Substitute 6 for x.

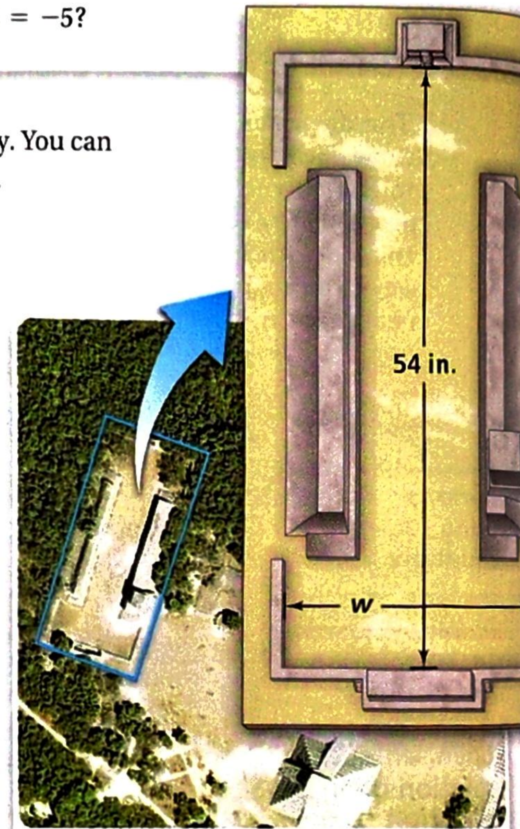
$32 \neq 24$ Simplify.

No, $x = 6$ is not a solution of the equation $32 = 2x + 12$.

Got It? **2.** Is $m = \frac{1}{2}$ a solution of the equation $6m - 8 = -5$?

In real-world problems, the word *is* can indicate equality. You can represent some real-world situations using an equation.

Problem 3 Writing an Equation



Satellite view of Chichén Itzá

Multiple Choice **An art student wants to make a model of the Mayan Great Ball Court in Chichén Itzá, Mexico. The length of the court is 2.4 times its width. The length of the student's model is 54 in. What should the width of the model be?**

Ⓐ 2.4 in. Ⓒ 22.5 in.

Ⓑ 11.25 in. Ⓓ 129.6 in.

Plan

Why do you need to test each answer choice?
You should test each answer choice in case you made a calculation error. If you get two correct answers, then you know you need to double-check your work.

Relate	The length	is	2.4	times	the width
Define	Let	w	=	the width of the model.	
Write	54	=	2.4	·	w

Test each answer choice in the equation to see if it is a solution.

Check A:	**Check B:**	**Check C:**	**Check D:**
$54 = 2.4w$	$54 = 2.4w$	$54 = 2.4w$	$54 = 2.4w$
$54 \stackrel{?}{=} 2.4(2.4)$	$54 \stackrel{?}{=} 2.4(11.25)$	$54 \stackrel{?}{=} 2.4(22.5)$	$54 \stackrel{?}{=} 2.4(129.6)$
$54 \neq 5.76$	$54 \neq 27$	$54 = 54$ ✔	$54 \neq 311.04$

The correct answer is C.

Got It? **3.** The length of the ball court at La Venta is 14 times the height of its walls. Write an equation that can be used to find the height of a model that has a length of 49 cm.

Problem 4 Using Mental Math to Find Solutions

Plan

How can you find the solution of an equation?
You can use mental math to find a value that makes the equation true.

What is the solution of each equation? Use mental math.

	Think	Solution	Check
A $x + 8 = 12$	What number plus 8 equals 12?	4	$4 + 8 = 12$ ✔
B $\frac{a}{8} = 9$	What number divided by 8 equals 9?	72	$\frac{72}{8} = 9$ ✔

Got It? **4.** What is the solution of $12 - y = 3$? Use mental math.

Problem 5 Using a Table to Find a Solution

Think

How can you start?
You can use mental math to quickly check values like 0, 1, and 10. Use these results to choose a reasonable starting value for your table.

What is the solution of $5n + 8 = 48$? Use a table.

Make a table of values. Choose a starting value using mental math. $5(1) + 8 = 13$ and $5(10) + 8 = 58$, so 1 is too low and 10 is too high.

Try $n = 5$ and $n = 6$.

n	$5n + 8$	Value of $5n + 8$
5	$5(5) + 8$	33
6	$5(6) + 8$	38
7	$5(7) + 8$	43
8	$5(8) + 8$	48

The value of $5n + 8$ increases as n increases, so try greater values of n.

When $n = 8$, $5n + 8 = 48$. So the solution is 8.

Got It? **5. a.** What is the solution of $25 - 3p = 55$? Use a table.
b. What is a good starting value to solve part (a)? Explain your reasoning.

Problem 6 Estimating a Solution

What is an estimate of the solution of $-9x - 5 = 28$? Use a table.

To estimate the solution, find the integer values of x between which the solution must lie. $-9(0) - 5 = -5$ and $-9(1) - 5 = -14$. If you try greater values of x, the value of $-9x - 5$ gets farther from 28.

Think

Can identifying a pattern help you make an estimate?
Yes. Identify how the value of the expression changes as you substitute for the variable. Use the pattern you find to work *toward* the desired value.

Try lesser values, such as $x = -1$ and $x = -2$.

x	$-9x - 5$	Value of $-9x - 5$
-1	$-9(-1) - 5$	4
-2	$-9(-2) - 5$	13
-3	$-9(-3) - 5$	22
-4	$-9(-4) - 5$	31

Now the values of $-9x - 5$ are getting closer to 28.

28 is between 22 and 31, so the solution is between -3 and -4.

Got It? **6.** What is the solution of $3x + 3 = -22$? Use a table.

Lesson Check

Do you know HOW?

1. Is $y = -9$ a solution of $y + 1 = 8$?
2. What is the solution of $x - 3 = 12$? Use mental math.
3. **Reading** You can read 1.5 pages for every page your friend can read. Write an equation that relates the number of pages p that you can read and the number of pages n that your friend can read.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** Give an example of an equation that is true, an equation that is false, and an open equation.
5. **Open-Ended** Write an open equation using one variable and division.
6. **Compare and Contrast** Use two different methods to find the solution of the equation $x + 4 = 13$. Which method do you prefer? Explain.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Tell whether each equation is *true*, *false*, or *open*. Explain. See Problem 1.

7. $85 + (-10) = 95$
8. $225 \div t - 4 = 6.4$
9. $29 - 34 = -5$
10. $-8(-2) - 7 = 14 - 5$
11. $4(-4) \div (-8)6 = -3 + 5(3)$
12. $91 \div (-7) - 5 = 35 \div 7 + 3$
13. $4a - 3b = 21$
14. $14 + 7 + (-1) = 21$
15. $5x + 7 = 17$

Tell whether the given number is a solution of each equation. See Problem 2.

16. $8x + 5 = 29; 3$
17. $5b + 1 = 16; -3$
18. $6 = 2n - 8; 7$
19. $2 = 10 - 4y; 2$
20. $9a - (-72) = 0; -8$
21. $-6b + 5 = 1; \frac{1}{2}$
22. $7 + 16y = 11; \frac{1}{4}$
23. $14 = \frac{1}{3}x + 5; 27$
24. $\frac{3}{2}t + 2 = 4; \frac{2}{3}$

Write an equation for each sentence. See Problem 3.

25. The sum of $4x$ and -3 is 8.
26. The product of 9 and the sum of 6 and x is 1.
27. **Training** An athlete trains for 115 min each day for as many days as possible. Write an equation that relates the number of days d that the athlete spends training when the athlete trains for 690 min.
28. **Salary** The manager of a restaurant earns \$2.25 more each hour than the host of the restaurant. Write an equation that relates the amount h that the host earns each hour when the manager earns \$11.50 each hour.

Use mental math to find the solution of each equation. See Problem 4.

29. $x - 3 = 10$
30. $4 = 7 - y$
31. $18 + d = 24$
32. $2 - x = -5$
33. $\frac{m}{3} = 4$
34. $\frac{x}{7} = 5$
35. $6t = 36$
36. $20a = 100$
37. $13c = 26$

Use a table to find the solution of each equation. See Problem 5.

38. $2t - 1 = 11$ **39.** $5x + 3 = 23$ **40.** $0 = 4 + 2y$ **41.** $8a - 10 = 38$

42. $12 = 6 - 3b$ **43.** $8 - 5w = -12$ **44.** $-48 = -9 - 13n$ **45.** $\frac{1}{2}x - 5 = -1$

Use a table to find two consecutive integers between which the solution lies. See Problem 6.

46. $6x + 5 = 81$ **47.** $3.3 = 1.5 - 0.4y$ **48.** $-115b + 80 = -489$

B Apply

49. Bicycle Sales In the United States, the number y (in millions) of bicycles sold with wheel sizes of 20 in. or greater can be modeled by the equation $y = 0.3x + 15$, where x is the number of years since 1981. In what year were about 22 million bicycles sold?

50. Error Analysis A student checked whether $d = -2$ is a solution of $-3d + (-4) = 2$, as shown. Describe and correct the student's error.

$-3d + (-4) = 2$
$-3(-2) + (-4) \stackrel{?}{=} 2$
$-6 + (-4) \stackrel{?}{=} 2$
$-10 \neq 2$ ✗

51. Writing What are the differences between an expression and an equation? Does a mathematical expression have a solution? Explain.

52. Basketball A total of 1254 people attend a basketball team's championship game. There are six identical benches in the gymnasium. About how many people would you expect each bench to seat?

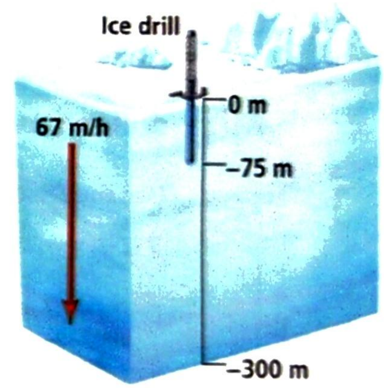
Find the solution of each equation using mental math or a table. If the solution lies between two consecutive integers, identify those integers.

53. $x + 4 = -2$ **54.** $4m + 1 = 9$ **55.** $10.5 = 3n - 1$ **56.** $-3 + t = 19$

57. $5a - 4 = -16$ **58.** $9 = 4 + (-y)$ **59.** $1 = -\frac{1}{4}n + 1$ **60.** $17 = 6 + 2x$

61. Open-Ended Give three examples of equations that involve multiplication and subtraction and have a solution of -4.

62. Think About a Plan Polar researchers drill into an ice sheet. The drill is below the surface at the location shown. The drill advances at a rate of 67 m/h. About how many hours will it take the drill to reach a depth of 300 m?
- What equation models this situation?
- What integers do you need?



63. Deliveries The equation $25 + 0.25p = c$ gives the cost c in dollars that a store charges to deliver an appliance that weighs p pounds. Use the equation and a table to find the weight of an appliance that costs \$55 to deliver.

64. Look for a Pattern Use a table. Evaluate $2x + 2$ for $x = -2, -1, 0, 1, 2,$ and 3. What pattern do you notice in your results? Use this pattern to find the solution of $2x + 2 = 28$. Check your solution.

Challenge

65. **Reasoning** Your friend says that the solution of $15 = 4 + 2t$ is between two consecutive integers, because 15 is an odd number and 4 and 2 are both even numbers. Explain your friend's reasoning.

STEM 66. **Construction** A construction crew needs to install 550 ft of curbing along a street. The crew can install curbing at a rate of 32 ft/h. Yesterday the crew installed 272 ft of curbing. Today it wants to finish the job in at most 10 h, which includes a 15-min drive to the job, an hour lunch break, and 45 min to break down the equipment. Can the crew achieve its goal? Explain.

Standardized Test Prep

SAT/ACT

67. Which equation is false?

Ⓐ $\frac{2}{3} + 1 \cdot \frac{1}{2} = \frac{7}{6}$ Ⓑ $84 - 25 = 59$ Ⓒ $51 - (-57) = -6$ Ⓓ $3(-3) + 3 = -6$

68. Which equation has a solution of 4?

Ⓕ $0 = 8 + 2y$ Ⓖ $5x + 3 = 23$ Ⓗ $8a - 10 = 42$ Ⓘ $2t - 1 = 9$

69. At 7 P.M., the temperature is 6.8°C. Over the next 4 h, the temperature changes by the amounts shown in the table. What is the final temperature?

Ⓐ −12.6°C Ⓒ 1°C
Ⓑ 3.9°C Ⓓ 5.8°C

Temperature Changes

Time	Change in Temperature
8 P.M.	−0.4°C
9 P.M.	−1.2°C
10 P.M.	−1.3°C
11 P.M.	−2.9°C

70. Monique has ordered 32 pizzas to serve at the student government picnic. If each person will get $\frac{1}{4}$ of a pizza, how many people will she be able to serve?

Ⓕ 8 Ⓗ 64
Ⓖ 32 Ⓘ 128

Mixed Review

Use the Distributive Property to simplify each expression. See Lesson 1-7.

71. $7(4 + 2y)$ 72. $-6(3b + 11)$ 73. $(8 + 2t)(-2.1)$ 74. $(-1 + 5x)5$

Evaluate each expression for $m = 4$, $n = -1$, and $p = -\frac{1}{2}$. See Lesson 1-6.

75. $2m - 2n$ 76. $pm - n$ 77. $6mp$ 78. $7m \div (-4n)$
79. $8p - (-5n)$ 80. $-2m - n$ 81. $-1.5m \div 6p$ 82. $3n^2 \cdot (-10p^2)$

Get Ready! **To prepare for Lesson 1-9, do Exercises 83–86.**

Use a table to find the solution of each equation. See Lesson 1-8.

83. $4x - 1 = 7$ 84. $0 = 10 + 10y$ 85. $5\frac{1}{2} = 7 - \frac{1}{2}b$ 86. $3t - (-5.4) = 5.4$

Concept Byte

Use With Lesson 1-8

TECHNOLOGY

Using Tables to Solve Equations

Content Standard

Prepares for A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution . . .

MATHEMATICAL PRACTICES

You can solve equations by making a table using a graphing calculator.

Activity

A raft floats downriver at 9 mi/h. The distance y the raft travels can be modeled by the equation $y = 9x$, where x is the number of hours. Make a table on a graphing calculator to find how long it takes the raft to travel 153 mi.

Step1 Enter the equation $y = 9x$ into a graphing calculator.

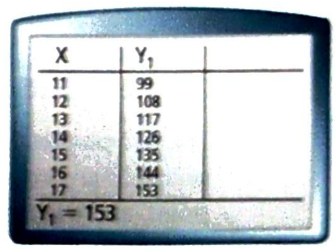
- Press y=. The cursor appears next to Y_1.
- Press 9 x,t,θ,n to enter $y = 9x$.

Step 2 Access the table setup feature.

- Press 2nd window.
- TblStart represents the starting value in the table. Enter 1 for TblStart.
- $\triangle$Tbl represents the change in the value of x as you go from row to row. Enter 1 for $\triangle$Tbl.

Step 3 Display the table and find the solution.

- Press 2nd graph. Use ▽ to scroll through the table until you find the x-value for which $y = 153$. This x-value is 17. It takes the raft 17 h to travel 153 mi.



Exercises

Solve each problem by making a table on a graphing calculator.

1. A town places 560 t of waste in a landfill each month. The amount y of waste in the landfill can be modeled by the equation $y = 560x$, where x is the number of months. How many months will it take to accumulate 11,200 t of waste in the landfill?

2. A coupon gives \$15 off a customer's purchase. The total amount y of the customer's purchase can be modeled by $y = x - 15$, where x is the amount of the purchase before the coupon is used. A customer using the coupon pays \$17 for a shirt. What was the original price of the shirt?

Review

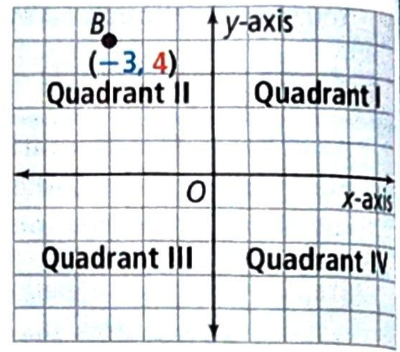
Use With Lesson 1-9

Graphing in the Coordinate Plane

Content Standard

Prepares for A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Two number lines that intersect at right angles form a **coordinate plane.** The horizontal axis is the ***x*-axis** and the vertical axis is the ***y*-axis.** The axes intersect at the **origin** and divide the coordinate plane into four sections called **quadrants.**



An **ordered pair** of numbers names the location of a point in the plane. These numbers are the **coordinates** of the point. Point B has coordinates $(-3, 4)$.

The first coordinate is the x-coordinate. $(-3, 4)$ The second coordinate is the y-coordinate.

To reach the point (x, y), you use the x-coordinate to tell how far to move right (positive) or left (negative) from the origin. You then use the y-coordinate to tell how far to move up (positive) or down (negative).

Activity

Play against a partner using two number cubes and a coordinate grid. One cube represents positive numbers and the other cube represents negative numbers.

- During each turn, a player rolls both cubes and adds the numbers to find an x-coordinate. Both cubes are rolled a second time, and the numbers are added to find the y-coordinate. The player graphs the resulting ordered pair on the grid.
- The two players take turns, with each player using a different color to graph points. If an ordered pair has already been graphed, the player does not graph a point, and the turn is over.
- Play ends after each player has completed 10 turns. The player with the most points graphed in a quadrant scores 1 for Quadrant I, 2 for Quadrant II, and so on. Points graphed on either axis do not count. If both players graph an equal number of points in a quadrant, both players score 0 for that quadrant.

Exercises

Describe a pair of number cube rolls that would result in a point plotted at the given location.

1. $(-3, 4)$ **2.** $(4, -3)$ **3.** in Quadrant III **4.** the origin

1-9 Patterns, Equations, and Graphs

Content Standards

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Also A.CED.2

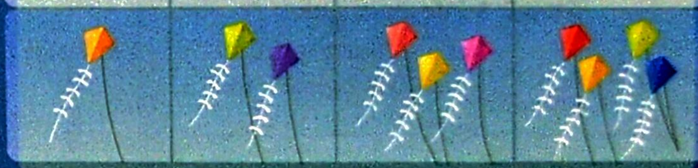
Objective To use tables, equations, and graphs to describe relationships

MATHEMATICAL PRACTICES

SOLVE IT! **Getting Ready!**

The table below shows the relationship between the number of kites in an arrangement and the total number of ribbons on the kites' tails. Describe the pattern in the table. How many kites could you make with 275 ribbons? Explain.

One Kite	Two Kites	Three Kites	Four Kites
5 Ribbons	10 Ribbons	15 Ribbons	20 Ribbons



Lesson Vocabulary
- solution of an equation
- inductive reasoning

In the Solve It, you may have described the pattern using words. You can also use an equation or a graph to describe a pattern.

Essential Understanding Sometimes the value of one quantity can be found if you know the value of another. You can represent the relationship between the quantities in different ways, including tables, equations, and graphs.

You can use an equation with two variables to represent the relationship between two varying quantities. A **solution of an equation** with two variables x and y is any ordered pair (x, y) that makes the equation true.

Plan

How can you tell whether an ordered pair is a solution? Replace x with the first value in the ordered pair and y with the second value in the ordered pair. Is the resulting equation true?

Problem 1 Identifying Solutions of a Two-Variable Equation

Is (3, 10) a solution of the equation $y = 4x$?

$y = 4x$

$10 \stackrel{?}{=} 4 \cdot 3$ Substitute 3 for x and 10 for y.

$10 \neq 12$ So, (3, 10) is not a solution of $y = 4x$.

Got It? 1. Is the ordered pair a solution of the equation $y = 4x$?

a. (5, 20) **b.** $(-5, -20)$ **c.** $(-20, -5)$ **d.** (1.5, 6)

You can represent the same relationship between two variables in several different ways.

Problem 2 Using a Table, an Equation, and a Graph

Ages Both Carrie and her sister Kim were born on October 25, but Kim was born 2 years before Carrie. How can you represent the relationship between Carrie's age and Kim's age in different ways?

Know
Kim was born 2 years before Carrie.

Need
Different ways to represent the relationship

Plan
Use a table, an equation, and a graph.

Step 1 Make a table.

Carrie's and Kim's Ages (years)

Carrie's Age	1	2	3	4	5	6	7	8	9	10
Kim's Age	3	4	5	6	7	8	9	10	11	12

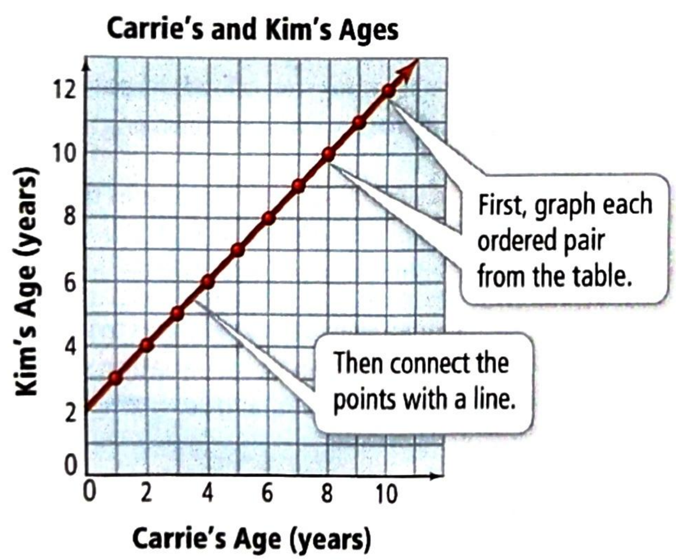
Step 2 Write an equation.

Let x = Carrie's age. Let y = Kim's age. From the table, you can see that y is always 2 greater than x.

So $y = x + 2$.

Think
Why does it make sense to connect the points on the graph?
A person's age can be any positive real number, and the ages of the girls are always 2 years apart. So every point on the line makes sense in this situation.

Step 3 Draw a graph.



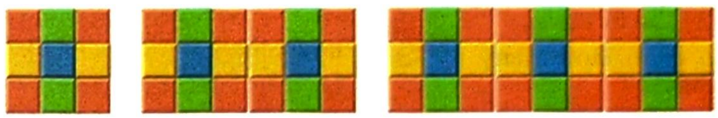
Got It? 2. a. Will runs 6 laps before Megan joins him at the track. They then run together at the same pace. How can you represent the relationship between the number of laps Will runs and the number of laps Megan runs in different ways? Use a table, an equation, and a graph.

b. Reasoning Describe how the graph in Problem 2 above would change if the difference in ages were 5 years instead of 2 years.

Inductive reasoning is the process of reaching a conclusion based on an observed pattern. You can use inductive reasoning to predict values.

Problem 3 Extending a Pattern

The table shows the relationship between the number of blue tiles and the total number of tiles in each figure. Extend the pattern. What is the total number of tiles in a figure with 8 blue tiles?



Tiles

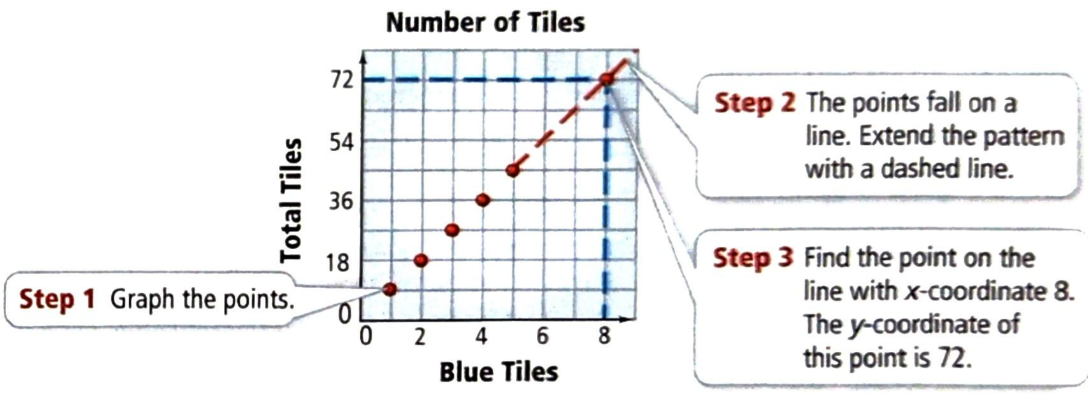
Number of Blue Tiles, x	Total Number of Tiles, y
1	9
2	18
3	27
4	36
5	45

Think

Should you connect the points on the graph with a solid line?

No. The number of tiles must be a whole number. Use a dotted line to see the trend.

Method 1 Draw a graph.



The total number of tiles is 72.

Method 2 Write an equation.

$y = 9x$	The total number of tiles is 9 times the number of blue tiles.
$= 9(8)$	Substitute 8 for x.
$= 72$	Simplify.

The total number of tiles is 72.

Got It? **3.** Use the tile figure from Problem 3.

a. Make a table showing the number of orange tiles and the total number of tiles in each figure. How many tiles in all will be in a figure with 24 orange tiles?

b. Make a table showing the number of blue tiles and the number of yellow tiles in each figure. How many yellow tiles will be in a figure with 24 blue tiles?

Lesson Check

Do you know HOW?

1. Is $(2, 4)$ a solution of the equation $y = x - 2$?
2. Is $(-3, -9)$ a solution of the equation $y = 3x$?
3. Drinks at the fair cost \$2.50. Use a table, an equation, and a graph to represent the relationship between the number of drinks bought and the cost.
4. **Exercise** On a treadmill, you burn 11 Cal in 1 min, 22 Cal in 2 min, 33 Cal in 3 min, and so on. How many Calories do you burn in 10 min?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Describe the difference between inductive reasoning and deductive reasoning.
6. **Compare and Contrast** How is writing an equation to represent a situation involving two variables similar to writing an equation to represent a situation involving only one variable? How are they different?
7. **Reasoning** Which of $(3, 5)$, $(4, 6)$, $(5, 7)$, and $(6, 8)$ are solutions of $y = x + 2$? What is the pattern in the solutions of $y = x + 2$?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Tell whether the given equation has the ordered pair as a solution. See Problem 1.

8. $y = x + 6; (0, 6)$
9. $y = 1 - x; (2, 1)$
10. $y = -x + 3; (4, 1)$
11. $y = 6x; (3, 16)$
12. $-x = y; (-3.1, 3.1)$
13. $y = -4x; (-2, 8)$
14. $y = x + \frac{2}{3}; \left(1, \frac{1}{3}\right)$
15. $y = x - \frac{3}{4}; \left(2, 1\frac{1}{4}\right)$
16. $\frac{x}{5} = y; (-10, -2)$

Use a table, an equation, and a graph to represent each relationship. See Problem 2.

17. Ty is 3 years younger than Bea.
18. The number of checkers is 24 times the number of checkerboards.
19. The number of triangles is $\frac{1}{3}$ the number of sides.
20. Gavin makes \$8.50 for each lawn he mows.

Use the table to draw a graph and answer the question. See Problem 3.

21. The table shows the height in inches of stacks of tires. Extend the pattern. What is the height of a stack of 7 tires?

Stacks of Tires

Number of Tires, x	Height of Stack, y
1	8
2	16
3	24
4	32

22. The table shows the length in centimeters of a scarf you are knitting. Suppose the pattern continues. How long is the scarf after 8 days?

Knitted Scarf

Number of Days, x	Length of Scarf, y
1	12.5
2	14.5
3	16.5
4	18.5

Use the table to write an equation and answer the question.

23. The table shows the heights in inches of trees after they have been planted. What is the height of a tree that is 64 in. tall in its pot?

Tree Height

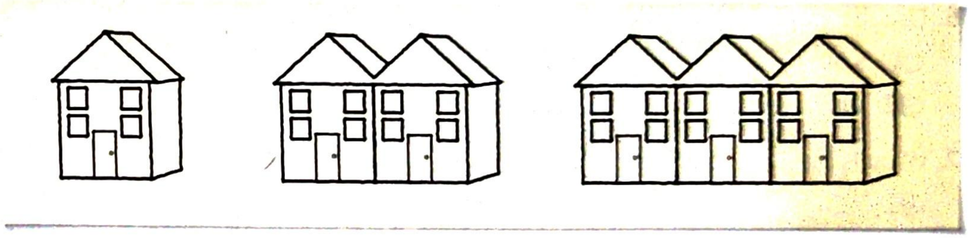
Height in Pot, x	Height Without Pot, y
30	18
36	24
42	30
48	36

24. The table shows amounts earned for pet sitting. How much is earned for a 9-day job?

Pet Sitting

Days, x	Dollars, y
1	17
2	34
3	51
4	68

Refer to the drawing of houses for Exercises 25 and 26.



25. Patterns Make a table and draw a graph to show the relationship between the number of houses and the number of windows.

a. What is the number of windows in 9 houses?

b. If n houses have k windows, write an expression to represent the number of windows for $n + 1$ houses.

26. Bobby says that a subdivision similar to the one above has a total of 202 windows. Is 202 a reasonable number of windows? Explain.

B Apply

Tell whether the given ordered pair is a solution of the equation.

27. $y = 2x + 7$; $(-2, 3)$

28. $-\frac{1}{4}x + 6 = y$; $(2, 4)$

29. $y = -1.2x - 2.6$; $(3.5, 6.8)$

30. Think About a Plan The table shows how long it takes Kayla to learn new songs. How many hours does Kayla need to practice to learn 9 songs?

- From row to row, how much does the number of hours h increase? How much does the number of songs s increase?
- By how many rows would you need to extend the table to solve the problem?

Kayla's Piano Practice

Hours, h	Songs Learned, s
1.5	1
3.0	2
4.5	3
6.0	4

31. Air Travel Use the table at the right. How long will the jet take to travel 5390 mi?

Passenger Jet Travel

Hours, h	1	2	3	4
Miles, m	490	980	1470	1960

x	y
0	−6
1	−5
2	−4
3	−3

32. Reasoning Savannah looks at the table shown and says the equation $y = x - 6$ represents the pattern. Mary says $y = x + (-6)$ represents the pattern. Who is correct? Explain.

33. Open-Ended Think of a real-world pattern. Describe the pattern using words and an equation with two variables. Define the variables.

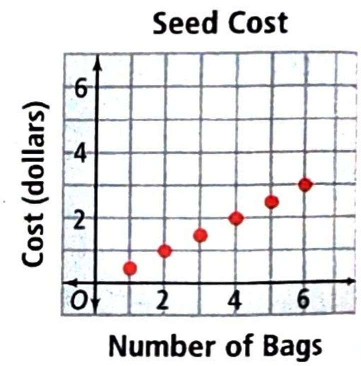
Challenge

34. Temperature Suppose the temperature starts at 60°F and rises 2°F every 45 min. Use a table, an equation, and a graph to describe the relationship between the amount of time that has passed in hours and the temperature.

35. Use a table, a graph, and an equation to represent the ordered pairs $(2, -5.5)$, $(-3, -0.5)$, $(1, -4.5)$, $(0, -3.5)$, $(-3.5, 0)$, and $(-1, -2.5)$.

Standardized Test Prep

SAT/ACT



36. Use the graph. What is the total price for 4 bags of seeds?

(A) \$.50 (C) \$4.00

(B) \$2.00 (D) \$8.00

37. What is the simplified form of the expression $-5(n - 2)$?

(F) $-7n$ (H) $-5n + 10$

(G) $-5n - 2$ (I) $n + 10$

38. If $a = 3$ and $b = -2$, what does $-2b - a$ equal?

(A) −9 (B) −7 (C) −1 (D) 1

39. What is the value of -3^4?

(F) −81 (G) −12 (H) 12 (I) 81

Mixed Review

Tell whether the given number is a solution of each equation. See Lesson 1-8

40. $3x + 7 = 10; 0$ **41.** $80 = 4a; 20$ **42.** $10 = -5t; -2$

Give an example that illustrates each property. See Lesson 1-4

43. Commutative Property of Addition

44. Associative Property of Multiplication

45. Identity Property of Multiplication

46. Zero Property of Addition

Get Ready! **To prepare for Lesson 2-1, do Exercises 47–54.**

See Lesson 1-5

Find each sum or difference.

47. $12 + (-3)$ **48.** $-7 + 4$ **49.** $-8 + (-6)$ **50.** $-42 + 15$

51. $32 - (-8)$ **52.** $-18 - 12$ **53.** $-15 - (-14)$ **54.** $-76 - 5$

1

Pull It All Together ASSESSMENT

To solve these problems, you will pull together many concepts and skills related to expressions, real-number properties and operations, and equations.

BIG idea Variable

You can use variables to represent quantities that are unknown or vary and to write expressions and equations.

Performance Task 1

Solve. Show all of your work and explain your steps.

A riding-stable manager is planning a nutritional diet for 6 horses. The manager finds the table below in a guide about horse health. The cost of 1000 Calories of horse feed is \$.15.

a. What is an expression for the total cost of feeding h horses?

b. Explain why your expression will give the total cost. Use a number for h as you explain your expression.

Calories Needed				
Number of Horses	1	2	3	4
Daily Calories Needed	15,000	30,000	45,000	60,000

Performance Task 2

Use the table at the right to complete each part.

a. Copy the table. Extend the table by writing expressions for y when $x = 5, 6,$ and 7.

b. Write an equation that relates x and y. Use your equation to find the value of y when $x = 15$.

x	y
1	7
2	9
3	11
4	13

BIG idea Properties

The properties of real numbers describe relationships that are always true. The properties of real numbers are true in both arithmetic and algebra. You can use them to rewrite expressions.

Performance Task 3

Solve. Show all of your work and explain your steps.

You are buying gifts for 10 people. You decide to buy each person either a CD or a DVD. A CD costs \$12 and a DVD costs \$20.

a. Let $c =$ the number of CDs you decide to buy. What is an expression in terms of c for the number of DVDs you buy?

b. What is an expression in terms of c for the cost of the CDs? For the cost of the DVDs?

c. Write and simplify an expression in terms of c for the *total* cost of all the gifts you buy. What properties of real numbers did you use to simplify the expression?

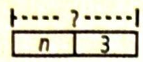
1 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Variable
You can use variables to represent quantities and to write algebraic expressions and equations.

2 Properties
The properties of real numbers describe relationships that are always true. You can use them to rewrite expressions.

Variables and Expressions (Lesson 1-1)
a number n plus 3
$n + 3$



Operations With Real Numbers (Lessons 1-2, 1-5, and 1-6)
2^5 $0 \cdot 3$ $2 + (-5)$ $7(-3)$

Properties (Lessons 1-4 and 1-7)
$a \cdot b = b \cdot a$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
$a(b + c) = ab + ac$

Patterns and Equations (Lessons 1-8 and 1-9)
1 variable: $x + 3 = 5$
2 variables: $y = 2x$

Chapter Vocabulary

- absolute value (p. 31)
- additive inverse (p. 32)
- algebraic expression (p. 4)
- base (p. 10)
- coefficient (p. 48)
- constant (p. 48)
- counterexample (p. 25)
- deductive reasoning (p. 25)
- Distributive Property (p. 46)
- element of the set (p. 17)
- equation (p. 53)
- equivalent expressions (p. 23)
- evaluate (p. 12)
- exponent (p. 10)
- inductive reasoning (p. 63)
- inequality (p. 19)
- integer (p. 18)
- irrational number (p. 18)
- like terms (p. 48)
- multiplicative inverse (p. 40)
- natural number (p. 18)
- numerical expression (p. 4)
- open sentence (p. 53)
- opposite (p. 32)
- order of operations (p. 11)
- perfect square (p. 17)
- power (p. 10)
- quantity (p. 4)
- radical (p. 16)
- radicand (p. 16)
- rational number (p. 18)
- real number (p. 18)
- reciprocal (p. 41)
- set (p. 17)
- simplify (p. 10)
- solution of an equation (p. 54, 61)
- square root (p. 16)
- subset (p. 17)
- term (p. 48)
- variable (p. 4)
- whole number (p. 18)

Choose the correct term to complete each sentence.

1. Real numbers that you cannot represent as a quotient of two integers are _?_ numbers.

2. The sum of a number and its _?_ equals zero.

3. You can simplify an expression by combining _?_.

4. _?_ is a number's distance from zero on a number line.

5. When you make conclusions based on patterns you observe, you use _?_.

1-1 Variables and Expressions

Quick Review

A **variable** is a symbol, usually a letter, that represents values of a variable quantity. For example, d often represents distance. An **algebraic expression** is a mathematical phrase that includes one or more variables. A **numerical expression** is a mathematical phrase involving numbers and operation symbols, but no variables.

Example

What is an algebraic expression for the word phrase *3 less than half a number x*?

You can represent "half a number x" as $\frac{x}{2}$. Then subtract 3 to get $\frac{x}{2} - 3$.

Exercises

Write an algebraic expression for each word phrase.

6. the product of a number w and 737

7. the difference of a number q and 8

8. the sum of a number x and 84

9. 9 more than the product of 51 and a number t

10. 14 less than the quotient of 63 and a number h

11. a number b less the quotient of a number k and 5

Write a word phrase for each algebraic expression.

12. $12 + a$

13. $r - 31$

14. $19t$

15. $b \div 3$

16. $7c - 3$

17. $2 + \frac{x}{8}$

18. $\frac{y}{11} - 6$

19. $21d + 13$

1-2 Order of Operations and Evaluating Expressions

Quick Review

To **evaluate** an algebraic expression, first substitute a given number for each variable. Then simplify the numerical expression using the order of operations.

1. Do operation(s) inside grouping symbols.
2. Simplify powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Example

A student studies with a tutor for 1 hour each week and studies alone for h hours each week. What is an expression for the total hours spent studying each week? Evaluate the expression for $h = 5$.

The expression is $h + 1$. To evaluate the expression for $h = 5$, substitute 5 for h: $(5) + 1 = 6$.

Exercises

Simplify each expression.

20. 9^2

21. 5^3

22. $\left(\frac{1}{6}\right)^2$

23. $7^2 \div 5$

24. $(2^4 - 6)^2$

25. $(3^3 - 4) + 5^2$

Evaluate each expression for $c = 3$ and $d = 5$.

26. $d^3 \div 15$

27. $(2 + d)^2 - 3^2$

28. $cd^2 + 4$

29. $(3c^2 - 3d)^2 - 21$

30. The expression $6s^2$ represents the surface area of a cube with edges of length s.

a. What is the cube's surface area when $s = 6$?

b. **Reasoning** Explain how a cube's surface area changes if you divide s by 2 in the expression $6s^2$.

31. A race car travels at 205 mi/h. How far does the car travel in 3 h?

1-3 Real Numbers and the Number Line

Quick Review

The rational numbers and irrational numbers form the **set** of **real numbers.**

A **rational number** is any number that you can write as $\frac{a}{b}$, where a and b are integers and $b \neq 0$. The rational numbers include all positive and negative integers, as well as fractions, mixed numbers, and terminating and repeating decimals.

Irrational numbers cannot be represented as the quotient of two integers. They include the square roots of all positive integers that are not perfect squares.

Example

Is the number rational or irrational?

A -5.422	rational
B $\sqrt{7}$	irrational

Exercises

Tell whether each number is rational or irrational.

32. π **33.** $-\frac{1}{2}$

34. $\sqrt{\frac{2}{3}}$ **35.** $0.\overline{57}$

Estimate each square root. Round to the nearest integer.

36. $\sqrt{99}$ **37.** $\sqrt{48}$ **38.** $\sqrt{30}$

Name the subset(s) of the real numbers to which each number belongs.

39. -17 **40.** $\frac{13}{62}$ **41.** $\sqrt{94}$

42. $\sqrt{100}$ **43.** 4.288 **44.** $1\frac{2}{3}$

Order the numbers in each exercise from least to greatest.

45. $-1\frac{2}{3}, 1.6, -1\frac{4}{5}$ **46.** $\frac{7}{9}, -0.8, \sqrt{3}$

1-4 Properties of Real Numbers

Quick Review

You can use properties such as the ones below to simplify and evaluate expressions.

Commutative Properties $-2 + 7 = 7 + (-2)$

$3 \times 4 = 4 \times 3$

Associative Properties $2 \times (14 \times 3) = (2 \times 14) \times 3$

$3 + (12 + 2) = (3 + 12) + 2$

Identity Properties $-6 + 0 = -6$

$21 \times 1 = 21$

Zero Property of Multiplication $-7 \times 0 = 0$

Multiplication Property of −1 $6 \cdot (-1) = -6$

Example

Use an identity property to simplify $-\frac{7ab}{a}$.

$-\frac{7ab}{a} = -7b \cdot \frac{a}{a} = -7b \cdot 1 = -7b$

Exercises

Simplify each expression. Justify each step.

47. $-8 + 9w + (-23)$

48. $\frac{6}{5} \cdot (-10 \cdot 8)$

49. $\left(\frac{4}{3} \cdot 0\right) \cdot (-20)$

50. $53 + (-12) + (-4t)$

51. $\frac{6 + 3}{9}$

Tell whether the expressions in each pair are equivalent.

52. $(5 - 2)c$ and $c \cdot 3$

53. $41 + z + 9$ and $41 \cdot z \cdot 9$

54. $\frac{81xy}{3x}$ and $9xy$

55. $\frac{11t}{(5 + 7 - 11)}$ and t

1-5 and 1-6 Operations With Real Numbers

Quick Review

To add numbers with different signs, find the difference of their **absolute values.** Then use the sign of the addend with the greater absolute value.

$3 + (-4) = -(4 - 3) = -1$

To subtract, add the opposite.

$9 - (-5) = 9 + 5 = 14$

The product or quotient of two numbers with the same sign is positive: $5 \cdot 5 = 25$ $\quad (-5) \cdot (-5) = 25$

The product or quotient of two numbers with different signs is negative: $6 \cdot (-6) = -36$ $\quad -36 \div 6 = -6$

Example

Cave explorers descend to a site that has an elevation of -1.3 km. (Negative elevation means below sea level.) The explorers descend another 0.6 km before they stop to rest. What is the elevation at their resting point?

$-1.3 + (-0.6) = -1.9$

The elevation at their resting point is -1.9 km.

Exercises

Find each sum. Use a number line.

56. $1 + 4$ **57.** $3 + (-8)$ **58.** $-2 + (-7)$

Simplify each expression.

59. $-5.6 + 7.4$ **60.** -12^2

61. $-5(-8)$ **62.** $4.5 \div (-1.5)$

63. $-13 + (-6)$ **64.** $-9 - (-12)$

65. $(-2)(-2)(-2)$ **66.** $-54 \div (-0.9)$

Evaluate each expression for $p = 5$ and $q = -3$.

67. $-3q + 7$ **68.** $-(4q)$

69. $q - 8$ **70.** $5p - 6$

71. $-(2p)^2$ **72.** $7q - 7p$

73. $(pq)^2$ **74.** $2q \div (4p)$

1-7 The Distributive Property

Quick Review

Terms with exactly the same variable factors are **like terms.** You can combine like terms and use the Distributive Property to simplify expressions.

Distributive Property $\quad a(b + c) = ab + ac$

$a(b - c) = ab - ac$

Example

Simplify $7t + (3 - 4t)$.

$7t + (3 - 4t)$	$= 7t + (-4t + 3)$	Commutative Property
	$= (7t + (-4t)) + 3$	Associative Property
	$= (7 + (-4))t + 3$	Distributive Property
	$= 3t + 3$	Simplify.

Exercises

Simplify each expression.

75. $5(2x - 3)$ **76.** $-2(7 - a)$

77. $(-j + 8)\frac{1}{2}$ **78.** $3v^2 - 2v^2$

79. $2(3y - 3)$ **80.** $(6y - 1)\frac{1}{4}$

81. $(24 - 24y)\frac{1}{4}$ **82.** $6y - 3 - 5y$

83. $\frac{1}{3}y + 6 - \frac{2}{3}y$ **84.** $-ab^2 - ab^2$

85. **Music** All 95 members of the jazz club pay \$30 each to go see a jazz performance. What is the total cost of tickets? Use mental math.

86. **Reasoning** Are $8x^2y$ and $-5yx^2$ like terms? Explain.

1-8 An Introduction to Equations

Quick Review

An **equation** can be true or false, or it can be an **open sentence** with a variable. A **solution** of an equation is the value (or values) of the variable that makes the equation true.

Example

Is $c = 6$ a solution of the equation $25 = 3c - 2$?

$25 = 3c - 2$

$25 \stackrel{?}{=} 3 \cdot 6 - 2$ Substitute 6 for c.

$25 \neq 16$ Simplify.

No, $c = 6$ is not a solution of the equation $25 = 3c - 2$.

Exercises

Tell whether the given number is a solution of each equation.

87. $17 = 37 + 4f; f = -5$

88. $-3a^2 = 27; a = 3$

89. $3b - 9 = 21; b = -10$

90. $-2b + 4 = 3; b = \frac{1}{2}$

Use a table to find or estimate the solution of each equation.

91. $x + (-2) = 8$

92. $3m - 13 = 24$

93. $4t - 2 = 9$

94. $6b - 3 = 17$

1-9 Patterns, Equations, and Graphs

Quick Review

You can represent the relationship between two varying quantities in different ways, including tables, equations, and graphs. A **solution of an equation** with two variables is an **ordered pair** (x, y) that makes the equation true.

Example

Bo makes $15 more per week than Sue. How can you represent this with an equation and a table?

First write an equation. Let b = Bo's earnings and s = Sue's earnings. Bo makes $15 more than Sue, so $b = s + 15$. You can use the equation to make a table for $s = 25, 50, 75,$ and 100.

Sue's Earnings (s)	25	50	75	100
Bo's Earnings (b)	40	65	90	115

Exercises

Tell whether the given ordered pair is a solution of each equation.

95. $3x + 5 = y; (1, 8)$

96. $y = -2(x + 3); (-6, 0)$

97. $y = (x - 1.2)(-3); (0, 1.2)$

98. $10 - 5x = y; (-4, 10)$

99. Describe the pattern in the table using words, an equation, and a graph. Extend the pattern for $x = 5, 6,$ and 7.

x	y
1	15
2	25
3	35
4	45

Chapter Test

Do you know HOW?

1. Write an algebraic expression for the phrase *the quotient of n and 6.*

2. Write a word phrase for $-12t + 2$.

3. Evaluate the expression $-(pq)^2 \div (-8)$ for $p = 2$ and $q = 4$.

4. **Dance** The table shows how the total cost of dance classes at a studio depends on the number of classes you take. Write a rule in words and as an algebraic expression to model the relationship.

Dance Classes

Number of Classes	Total Cost
1	$(1 \times 15) + 20$
2	$(2 \times 15) + 20$
3	$(3 \times 15) + 20$

Simplify each expression.

5. $-20 - (-5) \cdot (-2^2)$

6. $\left(-\frac{1}{4}\right)^3$

7. $-\frac{7ab}{a}$, $a \neq 0$

8. $-|-25|$

9. $\sqrt{\frac{16}{25}}$

10. Is each statement true or false? If false, give a counterexample.
    - **a.** For all real numbers a and b, $a \cdot b$ is equivalent to $b \cdot a$.
    - **b.** For all real numbers a and b, $a(b \cdot c) = ab \cdot ac$

11. Is the ordered pair $(2, -5)$ a solution to the equation $4 + 3x = -2y$? Show your work.

12. Order the numbers $-\frac{7}{8}, \frac{7}{4}, -1\frac{4}{5}$, and $-\frac{13}{16}$ from least to greatest.

13. **Soccer** There are t teams in a soccer league. Each team has 11 players. Make a table, write an equation, and draw a graph to describe the total number of players p in the league. How many players are on 17 teams?

Simplify each expression.

14. $5x^2 - x^2$

15. $12 \div \left(-\frac{3}{4}\right)$

16. $-(-2 + 6t)$

17. $-3[b - (-7)]$

18. Name the subset(s) of the real numbers to which each number belongs.
    - **a.** -2.324
    - **b.** $\sqrt{46}$

19. Identify each property.
    - **a.** $a(b + c) = ab + ac$
    - **b.** $(a + b) + c = a + (b + c)$

Do you UNDERSTAND?

20. Is the set of positive integers the same as the set of nonnegative integers? Explain.

21. **Error Analysis** Find and correct the error in the work shown at the right.

$$-3 \cdot 5 + -9 \div 4 + 3^2$$
$$-15 + -9 \div 4 + 9$$
$$-24 \div 4 + 9$$
$$-6 + 9$$
$$3$$

22. Is the following statement true or false? If the product of three numbers is negative, then all the numbers are negative. If false, give a counterexample.

23. **Reasoning** You notice that 10°C = 50°F, 20°C = 68°F, and 30°C = 86°F. Use inductive reasoning to predict the value in degrees Fahrenheit of 40°C.

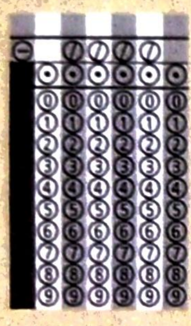
24. **Reasoning** When is the absolute value of a difference equal to the difference of the absolute values? Explain.

1 Cumulative Standards Review

ASSESSMENT

TIPS FOR SUCCESS

Some test questions ask you to enter a numerical answer on a grid. In this textbook, you will record answers on a grid like the one shown below.



TIP 1

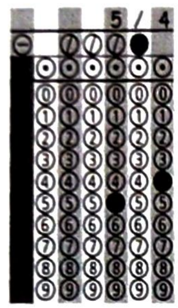
An answer may be either a fraction or a decimal. If an answer is a mixed number, rewrite it as an improper fraction or as a decimal.

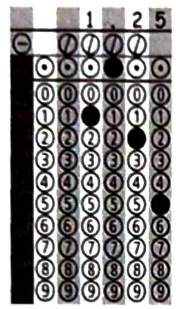
What is the value of $\frac{1}{2} + \frac{3}{4}$?

Solution

$\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$

Record the answer as $\frac{5}{4}$ or 1.25.





Do not record the answer as $1\frac{1}{4}$ because the test-scoring computer will read it as $\frac{11}{4}$.

TIP 2

You do not have to simplify a fraction unless the question asks for simplest form or the fraction does not fit on the grid.

Think It Through

You can add the fractions as shown in the solution on the left, or you can convert the fractions to decimals and add.

$\frac{1}{2} + \frac{3}{4} = 0.5 + 0.75 = 1.25$

Record the decimal answer on the grid.

Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Select the correct term to complete each sentence.

A. An algebraic (*expression, equation*) is a mathematical sentence with an equal sign.

B. A (*coefficient, constant*) is a numerical factor of a term.

C. The (*exponent, base*) of a power tells how many times a number is used as a factor.

D. To (*simplify, evaluate*) an algebraic expression, you substitute a given number for each variable.

E. A(n) (*rational, irrational*) number is any number that you can write in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$.

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. Which expression shows the product of z and $x - y$?

   (A) $z(x - y)$ (C) $x(z - y) + x$

   (B) $z \div x - y$ (D) $\frac{z}{x} - xy$

2. What is the solution of the equation $7d + 7 = 14$?

   (F) -3 (H) 1

   (G) -1 (I) 3

3. What is the value of the expression $8(-9) - 6(-3)$?

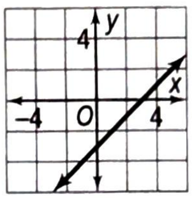
   (A) -90 (C) 54

   (B) -54 (D) 90

4. You ship an 8-lb care package to your friend at college. It will cost you \$.85 per pound, plus a flat fee of \$12, to ship the package. Which equation can you use to find the total cost C?

F $C = (8 \cdot 12) + 0.85$
H $C = 8 \cdot (0.85 + 12)$
G $C = (8 + 0.85) \cdot 12$
I $C = (8 \cdot 0.85) + 12$

5. The graph of $y = x - 3$ is shown below. Which ordered pair is NOT a solution of the equation $y = x - 3$?



A $(-4, -7)$
C $(0, -3)$
B $(12, 9)$
D $(-8, 11)$

6. Which property does the equation $4 + x + 7 = 4 + 7 + x$ illustrate?

F Identity Property of Addition
G Distributive Property
H Commutative Property of Addition
I Associative Property of Addition

7. Bill has a \$10 coupon for a party store. He needs to buy some balloons for a birthday party. If each balloon costs \$2 and Bill uses his coupon, what is an equation that gives the total price y of his purchase?

A $y = 2x$
B $y = 2x - 10$
C $y = 2x + 10$
D $y = 10 - 2x$

8. You own 100 shares of Stock A and 30 shares of Stock B. On Monday, Stock A decreased by \$.40 per share and Stock B increased by \$.25 per share. Which equation can be used to find the total change in value of your shares?

F $V = (100 \cdot -0.40) + (30 \cdot 0.25)$
G $V = (100 \cdot -0.40) + (30 \cdot -0.25)$
H $V = (100 \cdot 0.40) + (30 \cdot 0.25)$
I $V = (100 \cdot 0.40) + (30 \cdot -0.25)$

9. The table shows the relationship between the number of laps x you swim in a pool and the distance y, in meters, that you swim. Which equation describes the pattern in the table?

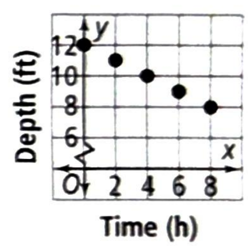
Laps, x	2	4	5	8
Distance (m), y	100	200	250	400

A $y = 50x$
C $y = x + 50$
B $y = 100x$
D $y = x + 100$

10. A store is having a sale on cases of juice. The first two cases of juice cost \$8 each. Any additional cases of juice cost \$6 each. Which expression can be used to find the cost of buying 9 cases of juice?

F $(2 \cdot 8) + (9 \cdot 6)$
H $(2 \cdot 8) + (7 \cdot 6)$
G $(2 \cdot 6) + (7 \cdot 8)$
I $9 \cdot 14$

11. The graph below shows the depth of water in a leaking tank.



If the tank continues to leak at the same rate, what will be the depth of water after 10 hours?

A 5 ft
C 7 ft
B 6 ft
D 10 ft

12. Chris spends \$3 per square foot on carpet for a square room. If he spends about \$430 on carpet, what is the approximate length, in feet, of a side of the room?

F 12 ft
H 72 ft
G 36 ft
I 215 ft

13. What is an algebraic expression for *4 more than the product of 3 and a number x*?

A $(4 + 3)x$
C $4 - 3x$
B $4 + 3x$
D $3x - 4$

14. A blank CD can hold 80 min of music. You have burned m minutes of music onto the CD. Which equation models the amount of time t that is left on the CD?

(F) $t = 80 - m$ (H) $t = 80 + m$

(G) $t = m - 80$ (I) $t = 80m$

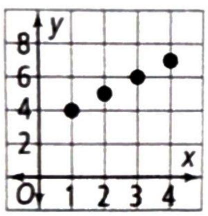
15. What is an expression for the sale price of a shirt that is sold at 40% off the original price p?

(A) $0.4p$ (C) $0.4p - p$

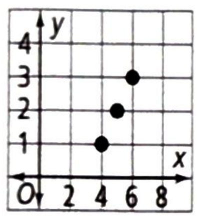
(B) $p + 0.4p$ (D) $p - 0.4p$

16. A clock originally costs x dollars. After you apply a \$3 discount to a clock that costs greater than \$3, the clock costs y dollars. Which graph models this situation?

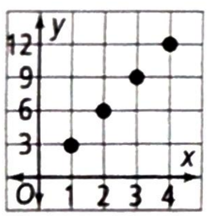
(F)



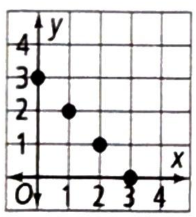
(H)



(G)



(I)



17. To order movie tickets online, John has to pay \$10 per ticket plus a \$2 handling fee for the whole order. Which equation can be used to determine the total cost C for t tickets?

(A) $C = 2t + 10$ (C) $C = 10t + 2$

(B) $C = 10t$ (D) $C = 12t$

18. The monthly cost C for a cell phone is \$20 each month plus \$.10 for each minute used. Which equation can be used to find the total cost (in dollars) for a month when m minutes are used?

(F) $C = m + 20$ (H) $C = 10m + 20$

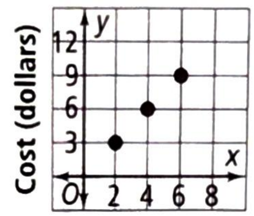
(G) $C = 0.1m + 20$ (I) $C = 20m$

GRIDDED RESPONSE

Record your answers in a grid.

19. Use the graph. What is the total cost, in dollars, for 8 bags of balloons?

Balloon Cost



Number of Bags

20. Cole has \$15 to spend on notebooks. Each notebook costs \$3.99. What is the greatest amount, in dollars, that Cole can spend on notebooks?

21. Simplify the expression $3ab + 4ab - 5ab$. What is the coefficient of the simplified expression?

22. Use mental math to find the solution of the equation $4t = 64$.

23. A student recorded the temperature change over time. The table below shows the results.

Time (h)	0.0	0.5	1.0	1.5
Temperature (°F)	76	73	70	67

If the temperature change continues at the same rate, what will the temperature be 3 hours after the start?

24. The formula $F = \frac{9}{5}C + 32$ changes a temperature reading from the Celsius scale C to the Fahrenheit scale F. What is the temperature measured in degrees Fahrenheit when the Celsius temperature is -5°C?

CHAPTER 2

Get Ready!

Lesson 1-4

Describing a Pattern

Describe the relationship shown in each table below using words and using an equation.

1.

Number of Lawns Mowed	Money Earned
1	\$7.50
2	\$15.00
3	\$22.50
4	\$30.00

2.

Number of Hours	Pages Read
1	30
2	60
3	90
4	120

Lesson 1-5

Adding and Subtracting Real Numbers

Simplify each expression.

3. $6 + (-3)$ **4.** $-4 - 6$ **5.** $-5 - (-13)$ **6.** $-7 + (-1)$

7. $-4.51 + 11.65$ **8.** $8.5 - (-7.9)$ **9.** $\frac{3}{10} - \frac{3}{4}$ **10.** $\frac{1}{5} + \left(-\frac{2}{3}\right)$

Lesson 1-6

Multiplying and Dividing Real Numbers

Simplify each expression.

11. $-85 \div (-5)$ **12.** $7\left(-\frac{6}{14}\right)$ **13.** $4^2(-6)^2$ **14.** $22 \div (-8)$

Lesson 1-7

Combining Like Terms

Simplify each expression.

15. $14k^2 - (-2k^2)$ **16.** $4xy + 9xy$ **17.** $6t + 2 - 4t$ **18.** $9x - 4 + 3x$

Looking Ahead Vocabulary

19. If you say that two shirts are *similar*, what does that mean about the shirts? What would you expect *similar* to mean if you are talking about two similar triangles?

20. A model ship is a type of *scale model*. What is the relationship of a model ship to the actual ship that it models?